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Shear capacity in concrete beams reinforced by stirrups with two different inclinations

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A B S T R A C T

A model for the estimation of shear capacity in Reinforced Concrete (RC) beams with web reinforcement is provided by introducing a generalization of classical plastic Nielsen's model, which is based on the variable-inclination stress-field approach. The proposed model is able to predict the shear capacity in RC beams reinforced by means of stirrups having two different inclinations and longitudinal web bars.

A numerical comparison with the results of experimental tests and those provided by a Finite Element Model (FEM) based on the well known theory of Modified Compression Field Theory (MCFT) is carried out for validating the robustness of the proposed model.

Finally, a set of parametrical analyses demonstrates the efficiency of the proposed double transverse-reinforcement system in enhancing the shear capacity of RC beams.

1. Introduction

In the last decades, many theoretical and experimental investigations have clarified several significant aspects of shear collapse in Reinforced Concrete (RC) and Prestressed Concrete (PC) elements [1–14]. The first model for shear strength prediction of RC beams, introduced by Ritter and Mörsch [15,16], was based on the truss analogy, where the contribution of the concrete is given by diagonal compression struts with a fixed 45° slope. In the last years, it has been replaced by a new model based on plastic theory [17–21]. In particular, this model assumes a compressive stress field in the concrete, and an equivalent uniformly distributed tensile stress field corresponding to the action of the stirrups. In this approach the inclination angle θ of the compressive stresses may be different from 45°. In fact, after yielding of web reinforcements, the inclination angle θ varies as the shear force increases. This approach is included in the models accepted in international codes [22,23] for the design of transversely-reinforced RC structural members subjected to shear.

The plastic model has also been adapted for the case of Fibre Reinforced Concrete (FRC) beams with or without stirrups [24–28].

Nowadays, a new way for the transverse reinforcement of RC beams in shear is gaining, attractiveness characterized by two different inclinations of shear reinforcement. This layout is being adopted in various structural typologies as: (a) in deep beams often used in bridges, reinforced with both stirrups inclined at two different angles (90° and 45°) and longitudinal reinforcement; (b) in semi-precast Hybrid Steel-Trussed Concrete Beams (HSTCB), consisting in a prefabricated steel truss embedded in a cast-in-situ concrete beam [29–32]. Moreover, the use of two different inclinations of shear reinforcement was very common in the design of beams in the past in RC frame. The upper longitudinal reinforcements in the region close to the beam to column joints were bent at 45° downwards, where they were not needed for bending moment anymore.

In the past codes, where the contribution due to concrete and steel reinforcement were additive, the contribution of multiple inclination of reinforcement could be easily taken into account by adding the contributions.

Currently, the design codes contain no specific provisions for the above-mentioned structural cases, and their design can be performed only by adjusting the existing models developed for other structural typologies.

For instance, referring to typology (b) and according to the recently-issued Italian guidelines on HSTCBs [33], the contribution of the transverse reinforcement exhibiting an inclination close to that of the concrete struts in compression should be neglected,

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and only the reinforcement in tension should be considered. As an alternative, the shear resistance may be evaluated either by enlarge the range of the inclination angle of the compressed concrete strut [30] or, again, by means of an additive approach, considering the steel truss as an additional resistant system associated to the remaining unreinforced concrete beam (shear capacity = capacity of the unreinforced concrete beam + capacity of the classical Mörsch truss) [32].

However, though sophisticated nonlinear FEM analyses, can be nowadays performed using accurate models [8,34–36], simplified mechanical simplified robust models are still needed, to speed up the design and to make the analysis of the various phases of the construction easier.

In this context, a physical model for the evaluation of the shear capacity in beams containing (a) two sets of stirrups with different inclinations, and (b) web longitudinal reinforcement is formulated by means of a suitable modification of a model proposed in previous paper [20,21,38], by extending the classical model currently proposed in Eurocode2 [22]. The model is validated by favorable comparison against the results of experimental tests on HSTCBs [32] and FEM analyses performed by using the FEM code VecTor2 [42] on traditional RC beams, because for this typology the authors have not found tests on RC beams with two orders of stirrups in the literature. The analyses demonstrate the efficiency of the proposed model in estimating the shear capacity.

2. Proposed model

The proposed model, aiming at evaluating the shear capacity of concrete beams reinforced with two differently-inclined series of stirrups, is based on the model derived in [20,21,38] where the following assumptions were made: (i) at the Ultimate Limit State (ULS), the resistant mechanism can be represented (Fig. 1) by: – two chords; the top compressed chord is made by the concrete and its reinforcement, the bottom tensile one made by the bottom longitudinal reinforcement as well as the prestressing reinforcement (if any); – and the web, carrying the shear action, made of concrete, longitudinal web reinforcement (if any), and the stirrups); (ii) both the stirrups and the longitudinal web reinforcement (if any) are subjected to a purely axial force (i.e. dowel action is considered elsewhere, as explained in the following); (iii) compared to the size of the structural members, the spacing of the stirrups and of the web longitudinal bars is so small that their actions can be modeled via different uniform stress fields; (iv) the concrete stress field in the web is inclined by the angle θ to the longitudinal axis, which may differ from $\beta \sim 45^\circ$ that is the alignment of the first cracks in a structural member subjected merely to bending and shear (like a beam at the Service Limit State SLS); the maximum shear capacity is achieved for $ctg\theta$ varying in the range $1 \leqslant ctg\theta \leqslant (ctg\theta)_{max}$ [39]; usually the value $(ctg\theta)_{max} = 2.5$ is assumed [22]; more severe limitation must be imposed in elements where flexural ductility is demanded [40]; (v) the constitutive laws of the materials are consistent with the theory of plasticity; (vi) the contributions to the shear capacity of dowel action, aggregate interlock are indirectly taken care of by introducing (through the angle θ) different orientations for the principal directions of the stress fields and the cracks; (vii) the contribution due to the tensile strength of concrete (V_c) is neglected; (viii) the arch action, which plays a remarkable role in the *D* (Disturbed) regions, is neglected; hence, the validity of the model is limited to *B* (Bernoulli) regions.

It has to be pointed out that according to [19], assumption (iv) may be used for beam with a transverse minimum shear reinforcement mechanical ratio of $0.16/f_c^{0.5}$ being f_c the concrete strength in compression.

The model is now extended to beams having two sets of web stirrups distributed along two different inclinations α_1 and α_2; they can be subjected either to compression or tension, depending on their inclination with respect to the longitudinal axis; thus internal actions in the web are modeled via four uniform stress fields, namely an horizontal one representing the longitudinal web reinforcement, the one representing the compressed concrete inclined by the angle θ and two representing the action of the two order of stirrups inclined by the angles α_1 and α_2, respectively (Fig. 1b).

The proposed model is formulated by applying the static theorem of the theory of plasticity, that makes it possible to evaluate the shear capacity of a beam via the so-called “lower-bound solution”.

In order to derive the equilibrium equations, the following notation is introduced: A_{tw1}, s_{tw1} and A_{tw2}, s_{tw2} are the areas of the cross-sections and the spaces of the reinforcement in the web with orientation α_1 and α_2 respectively; A_{lw} the area of the longitudinal reinforcement in the web; b_w and h the minimum web width and the depth of the cross section, respectively; f_{yd} and f'_{cd} the design steel strength and the reduced concrete strength in compression, respectively; hence, being A_{twi} the area of generic transverse reinforcement, the mechanical ratios ω_{twi} ($i = 1, 2$) are: $\omega_{twi} = A_{twi}/(b_w s_{twi} \sin\alpha_i) \times (f_{yd}/f'_{cd})$; likewise, A_{lw} is the area of the longitudinal reinforcement in the web and the mechanical ratio is $\omega_{lw} = A_{lw}/(b_w h) \times (f_{yd}/f'_{cd})$.

It has to be emphasized that, in order to take into account the biaxial stress state in the web, an “effectiveness” coefficient ν' ($\leqslant 1$) has been applied to the design compressive strength of the concrete f_{cd} for the concrete web stress field, namely $f'_{cd} = \nu' f_{cd}$ [22].

Aiming at evaluating the shear capacity of the beam, the following equilibrium equations of three different segments of the beam obtained by three different inclined sections parallel to either the direction of the concrete or one of the two orders of stirrups stress field are derived as shown in Appendix A:

$$\begin{aligned} v(x) = {} & \tilde{\sigma}_{tw1} \cdot \omega_{tw1} \cdot (ctg\theta + ctg\alpha_1) \cdot \sin^2\alpha_1 + \tilde{\sigma}_{tw2} \cdot \omega_{tw2} \\ & \cdot (ctg\theta + ctg\alpha_2) \cdot \sin^2\alpha_2 \end{aligned} \qquad (1)$$

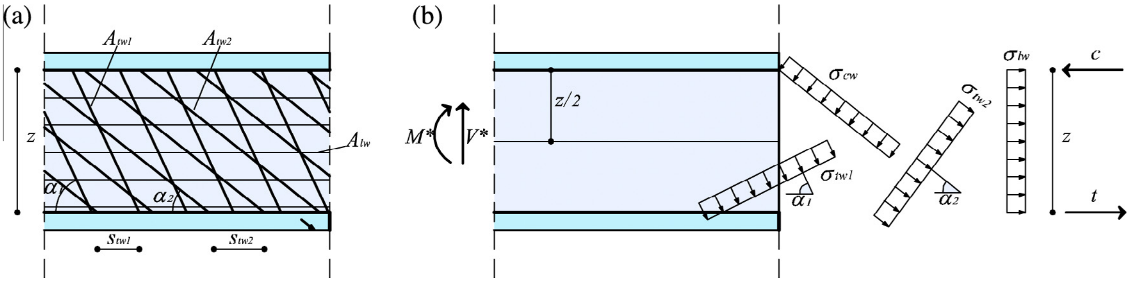

Fig. 1. Different types of reinforcement in a beam segment: (a) structural layout and (b) stress fields.