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# Effects of thermal environment on structural frequencies: Part II – A system identification model

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ABSTRACT

In civil structures such as bridges the changes in vibration characteristics such as modal frequencies are often used to detect damage for structural health monitoring. These vibration characteristics, however, change not only due to structural damage but also due to changes in the environmental conditions such as temperature variations. For structural health monitoring using modal frequencies, it is much desirable to be able to estimate the structural frequencies as a function of structural temperatures which can be conveniently measured. In this paper, we develop an approach to establish this modal frequency and bridge body temperature relationship. The subspace system identification-based approach, combined with input data filtering to capture trends introduced by seasonal changes in the temperature, is used to develop an approach to obtain this bridge body temperature–frequency relationship. Utilizing the simulated data for two types of bridge structures, the approach is exhaustively tested for its reproduction and generalization performances over different thermal regimes that are encountered in practice.

## 1. Introduction

The companion paper [1] presented the results of a simulation study where the impact of ever changing ambient thermal environment on the modal frequencies of the two common types of bridge superstructures was examined. The finite element approach was used to calculate the thermal responses and their effects on the structural frequencies for the environmental conditions recorded on a site in North Carolina, USA, over a period of several years. This long duration simulation confirmed the earlier field and experimental observations cited in the companion paper that the ambient environmental variations can significantly change the modal frequencies. For health monitoring purposes, it is much desirable to estimate this frequency variation from the recorded temperature data but without a detailed finite element analysis such as the one conducted in the companion paper.

The focus of this paper is, thus, to develop a model that can estimate the modal frequencies from the observed temperature values of a bridge structure. The estimation of frequencies directly in terms of the known temperature values has been a topic of much interest for structural health monitoring, especially for the bridge structures. Several earlier studies have explored this topic, utilizing short duration field data. A linear filter relating the spatial and temporal temperature distribution to modal frequency was proposed and tested [2] using the data collected on Almosa Canyon Bridge in New Mexico between July 27 and August 2, 1996 and tested on data collected on similar conditions between July 21 and 25, 1997. This was followed by a study [3] in which a combination of AR-ARX (Auto regressive – Auto regressive with Exogenous input) was used to extract damage sensitive features. The features were normalized with respect to temperature changes using Auto-associative Neural Network for damage identification. In another study [4], the authors used an ARX model to estimate the first four frequencies. A bilinear relationship between temperature and frequency with the transition around 0 °C was observed. For damage detection the magnitude of the differences between the estimated and recorded modal frequencies on one side of this transition point was used. Support vector machine approach has also been used [5,6] to develop a nonlinear frequency–temperature regression model. Both studies utilized the 770-h long temperature and frequency data recoded on Ting Kau Bridge, a multi-span cable-stayed bridge in Hong Kong. Alternate 385 points in the data were used for regression estimation and the leftover points for testing the model under similar thermal conditions. In the second study [6], the authors used the principal component analysis (PCA) to reduce the measurements needed for the model identification. The PCA and its nonlinear versions have also been effectively employed in several other studies to extract the environmental

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**Nomenclature**

| | |
|---|---|
| $x_k$ | system state vector at time $k$ |
| $y_k$ | system output vector at time $k$ |
| $A$ | system dynamics matrix |
| $B$ | input transform matrix |
| $C$ | output matrix |
| $D$ | direct feed through matrix |
| $w_k$ | uncertainties in input–output relation |
| $v_k$ | sensor noise |
| $y_{target}$ | measured output |
| $\alpha_i$ | parameters in regression model |
| $b_i$ | digital filter coefficients |

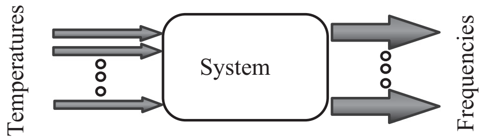

**Fig. 1.** Schematic model of the proposed identification problem.

effects from the modal features for damage detection [7,8] and damage identifications [9,10].

In this paper, we utilize the data generated in the companion paper [1] to develop and test a frequency estimation approach from the measured temperature values. The simulation data generated in the companion paper [1] covers a wide range of thermal conditions that bridges experience commonly, including the nonlinear temperature dependence of the material properties and thus the frequencies. These results showed that all modal frequencies have strong seasonal and diurnal trends, similar to those observed in environmental temperature variations. This observation has special significance for this study as these trends can be used in designing better numerical models that accommodates nonlinear temperature–frequency effects for more accurate predictions of system frequencies from the observed temperature data. To develop a temperature–frequency estimation approach that accommodates the nonlinear effects, we use the well-known linear state-space subspace system identification [11,12] technique with appropriate filtering of the observed trends from the data. The state-space model uses the recorded temperature variables as the input and provides the estimates of the modal frequencies as the output. The model is tested for robustness in different sets of data in non-overlapping seasons and limitations are identified. No damage detection studies are attempted at this stage (see Fig. 1).

## 2. Environmental effects: system identification

The data in the companion paper [1] consists of hourly temperature values at several locations on two bridge superstructures and corresponding modal frequencies calculated by the finite element analysis for the temperature, radiation and wind velocity values recorded at a site in North Carolina, USA over a period of several years. Although in actual practice the bridge temperature values will be appropriately measured at the site and the modal frequencies will be calculated by operational modal analysis of the vibration response, herein we use this comprehensive data to develop and test the models. We first develop the model using a part of the data set and then test its validity over another non-overlapping part of the data set. The step of developing the model is called as the "identification phase" whereas the validations step is called as the "simulation phase".

The following state-space model describes the subspace identification approach used to identify the relation between recorded temperature and frequencies;

$$\begin{aligned} &\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k + \mathbf{w}_k \\ &\mathbf{y}_k = C\mathbf{x}_k + D\mathbf{u}_k + \mathbf{v}_k \\ &with: \ \mathbf{x}_1 = \tilde{\mathbf{x}}_o \quad k = 1, 2, 3, \ldots \end{aligned} \tag{1}$$

where $\mathbf{u}$ is $(p \times 1)$ input temperature vector, $\mathbf{y}$ is $(q \times 1)$ output modal frequency vector, $\mathbf{x}$ is $(n \times 1)$ the state vector, $A$ is $(n \times n)$ system dynamics matrix, $B$ is $(n \times p)$ input transform matrix, $C$ is $(q \times n)$ output matrix and $D$ is $(q \times p)$ direct feed-through matrix. Here $p$ is the number of input temperature values, $q$ is the number of output frequencies and $n$ is the number of states or order of the model. The $(n \times 1)$ vector $w_k$ represents the process noise uncertainties in the input–output relation. The $(q \times 1)$ vector $v_k$ represents sensor noise, known as measurement-noise. The state vector and matrices are abstract quantities in the model and in general may not have any physical meaning. Eq. (1) implies that a linear model is assumed to represent the temperature–frequency relationship. However, since nonlinear temperature dependent material properties and the geometric stiffness are included, strictly speaking this relationship is not linear. If the nonlinear contributions are not strong, or can be filtered out, the assumption may provide acceptable results; however this acceptability must be tested by the accuracy achieved in the simulation phase.

In this study 'Numerical Algorithms for State-Space Subspace Identification' (N4SID) is used to identify the state-space model, the mathematical underpinning of which are given in references [11,12]. To define Eq. (1), number of inputs ($p$), number of outputs ($q$), model order ($n$) are to be specified and initial state vector ($\tilde{\mathbf{x}}_o$) is to be assumed or determined from the simulation data. N4SID

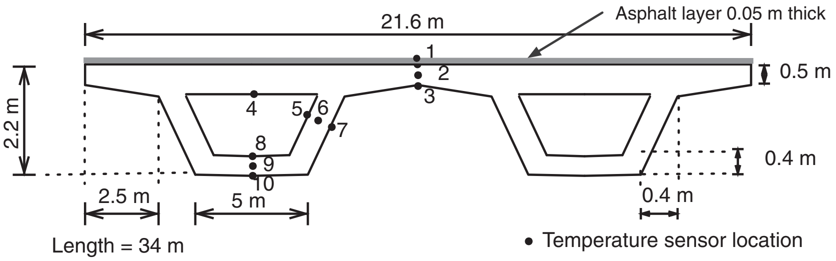

**Fig. 2.** Cross-sectional dimension of 34 m long bridge girder.

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