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Designing long-span steel girders by applying displacement control concepts

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ABSTRACT

In the present paper a new type of girder, suitable for covering long spans, is introduced. The load resisting mechanism of the system is based on the appropriate shape of the girder which is determined through a form-finding procedure. This optimal geometry is chosen in such a way so that no bending moments appear under external loading. Additional prestressed cables, integrated into the girder, are utilized as a means to limit vertical displacements, thus acting as a passive displacement control mechanism. At the same time, the externally simple supported structural scheme of the girder guarantees that no horizontal reactions are transferred to the columns. An application of this system as the main girder of an open roof structure is used to explain the proposed design method of the girders and to demonstrate the effectiveness of the system in covering long spans.

1. Introduction

The use of conventional structural systems in long-span structures with severe serviceability requirements can be quite challenging and hardly leads to cost-effective and viable solutions. Long-span structures tend to develop large bending moments under external loading which in turn lead to excessive deformations and vertical displacements. By applying concepts of displacement control in the design of a structure, however, its structural behaviour can be greatly enhanced [1,2]. In [3–7] prestressed cable systems are utilized as passive displacement control mechanisms for the moving loads in the design of bridges. In [8] even more passive control systems based on cables are proposed for long-span structural applications.

The system that is developed and examined in the present paper benefits from its appropriately chosen geometry as it works uniaxially in tension and compression and can be configured to remain undeformed under the permanent loading. In addition, a system of prestressed cables acts as a passive displacement control mechanism limiting excessive displacements. Similarly to classical shape optimization methods, the design procedure of the proposed system is concerned with the finding of the appropriate geometry that under a given set of loading conditions will maximize the stiffness. Unlike sophisticated optimization procedures ([9–11]) that involve advanced numerical methods to determine the absolute optimal solution of the structural system, a simplified approach that indirectly estimates the appropriate geometry has been adopted in the present study. The appropriate-optimal shape of the structure, together with the required prestress of the cables, is determined through a form-finding procedure that is based on the numerical treatment of a non-linear system of equations.

2. Description of the structural system

The proposed system is essentially a specially designed type of girder, ideal for long-spans, which can be used in conjunction with other conventional structural members such as purlins and horizontal bracings to form a roof-structure [8,12,13]. The system can also be used as the main girder of a beam-type bridge. Depending on the type of loading that is to be expected, and in particular on the magnitude of upward actions, two main variations of the system can be distinguished.

In the first variation which is suitable in the case of strong uplift forces, the girder has a lenticular type geometry that is formed by two slightly arched steel members of opposite curvature, connected at their ends (Fig. 1). The two chords are rigidly connected with each other throughout their length by a series of steel vertical struts which are positioned at the intersection points of the girder with the purlins and help transfer loading between the two chords. The girder is also equipped with a system of diagonal cables that connect the bottom and top part of consecutive struts. These cables are tensioned only to the point of not being slack and work as a stabilizing mechanism in the case of partial or non-uniform loading of

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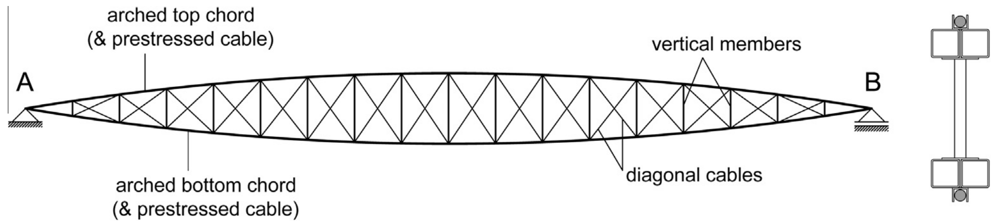

Fig. 1. Girder with top and bottom chord.

the girder. The response of the system can be further enhanced by the introduction of external prestressed cables adjacent to either chord of the girder. These cables are in contact with the chords, having the same geometry, and are anchored at the edges of the girder (Fig. 3). Their role is firstly to increase the stiffness of the structure and secondly due to the prestress to produce vertical reactions at the struts that counteract external loading.

In the case of small anticipated upward actions the system can be further modified by completely removing the bottom chord, thus significantly reducing the weight of the structure. In this variation of the system (Fig. 2), the girder consists only of an arched top chord which is supported from underneath by an external prestressed polygonal cable of negative curvature (concave facing upwards). The cable is anchored at the edges of the arched member and its polygonal shape is imposed by a series of vertical struts of appropriate length that are rigidly connected to the top chord. Like in the first variation of the girder, stabilizing diagonal cables connect the bottom and top part of successive struts. This system with the arched top chord and the polygonal prestressed bottom cable has been shown to perform well under downward actions in previous studies [12] and the present paper focuses on the analysis of the first variation of the girder.

A fundamental feature of the girder, essential in its design, is that it is externally simple supported and thus no horizontal actions are transferred to the columns or bearings (Fig. 4). Moreover, thanks to the appropriately chosen shape of the arched members, it is also achieved that the horizontal displacements at the roller of the girder caused by the moving loads are kept to a minimum level. It is worth noting, that the proposed structural system is equally effective in applications of inclined girders with large elevation difference at the supports.

3. Optimal prestress problem

For the analysis of the proposed system, the finite element method has been used within an optimal prestressing theoretical framework [1]. Let us consider a discretization of the girder structure and the external prestressed cables with n number of displacement degrees of freedom. Let $\mathbf{u}$ be the n-dimensional vector of displacements, $\mathbf{P}$ be the n-dimensional nodal load vector (energy equivalent to the displacements) and $\mathbf{K}$ be the $(n \times n)$-dimensional stiffness matrix of the structure. In general, the equilibrium of the system is then expressed by the following equation:

$$\mathbf{G} \cdot \boldsymbol{\sigma} = \mathbf{P} \quad (1)$$

The compatibility condition is given by the equation:

$$\mathbf{G}^T \cdot \mathbf{u} = \boldsymbol{\varepsilon} \quad (2)$$

In the above equations $\mathbf{G}$ is the $(n \times m)$-dimensional equilibrium matrix while $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ are the m-dimensional vectors of the generalized (internal) stresses and strains respectively.

Moreover, for a linear elastic material behaviour and accounting for initial strains $\boldsymbol{\varepsilon_0}$ the generalized stresses relate to the generalized displacements through the equation:

$$\boldsymbol{\sigma} = \mathbf{K_0} \cdot (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) \quad (3)$$

From relations Eqs. (1)–(3), after simple matrix manipulations the stiffness $\mathbf{K}$ of the structure can be written in the form:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{P} + \mathbf{G} \cdot \mathbf{K_0} \cdot \boldsymbol{\varepsilon}_0 \quad (4)$$

Let $\mathbf{z}$ be a q-dimensional control vector that is used to regulate the behaviour of the prestress control mechanism.

Let also, $\boldsymbol{\varepsilon_c}$ be the s-dimensional vector consisting of the discrete deformations of the passive control elements (i.e. the elongation of the cable segments) with $\boldsymbol{\varepsilon_{pr}}$ denoting their initial strain. The initial strain is used for control purposes and thus can be written as a function of the control vector $\mathbf{z}$, $\boldsymbol{\varepsilon_{pr}}(\mathbf{z})$.

The prestressing control action is modelled through the superposition of a set of variable nodal forces caused by the prestressed cables. The contribution of the prestressing forces is added to the loading vector $\mathbf{P}$ after appropriate transformation by a $(n \times s)$-dimensional transformation matrix $\mathbf{A}$. Thus, the enlarged loading vector $\mathbf{P}^*$ is applied to the structure:

$$\mathbf{P}^* = \mathbf{P} + \mathbf{A} \cdot \mathbf{K_c} \cdot \boldsymbol{\varepsilon_{pr}}(\mathbf{z}) \quad (5)$$

Here $\mathbf{K_c}$ denotes the $(s \times s)$-dimensional stiffness matrix of the discretized cable elements.

The passive control behaviour of the prestressed cables is introduced to the formulated problem through their constitutive relation in the form $\boldsymbol{\varepsilon_c} > \boldsymbol{\varepsilon_{pr}}$, which can be written as follows:

$$\boldsymbol{\varepsilon_{pr}}(\mathbf{z}) - \mathbf{A}^T \cdot \mathbf{u} \leqslant 0 \quad (6)$$

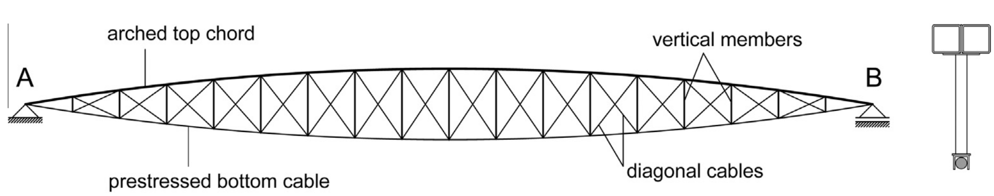

Fig. 2. Girder with top chord only.