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Analytical mode decomposition with Hilbert transform for modal parameter identification of buildings under ambient vibration

Zuocai Wang [a], Genda Chen [b,*]

[a] Department of Civil Engineering, Hefei University of Technology, Hefei 23009, China
[b] Department of Civil, Architectural, and Environmental Engineering, Missouri University of Science and Technology, Rolla, MO 65401, USA

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A B S T R A C T

A new analytical mode decomposition method in combination with the conventional random decrement technique is proposed for modal parameter identification under ambient vibration. The random decrement technique is used to extract the free vibration information from ambient vibration including closely-spaced modes. The analytical mode decomposition is developed with Hilbert transform to decompose the extracted free vibration with closely spaced natural frequencies into a series of modal responses from which modal parameters are evaluated. Emphasis in this study is placed on the characterization of frequency resolution, time duration effect, identification accuracy, and experimental validation of the new method. An energy error index is introduced and defined as the ratio between the squared modal response error and the exact modal energy over the response duration, accounting for the effects of both response amplitude and phase. Parametric studies with a 2-story building demonstrated a reduction of the energy error index from 88% with ambient vibration to 7.5% with 20-s free vibration, corresponding to a natural frequency space index of 0.033. The maximum error of the identified frequencies in all cases is less than 1%. At a frequency space index of 0.05, the energy error indices are less than 20% and 5% using 1-s and 7-s free vibration, respectively. The new method is then validated with shake table testing of a 3-story building frame installed with a tuned mass damper, and applied to a 36-story shear building with a 4-story light appendage with closely spaced modes. Both experiments and simulations showed high accuracy and effectiveness of the new method for building system identification from ambient vibration even when 5% noise.

1. Introduction

Modal parameter identification is a process to estimate the dynamic characteristics of a structural system from dynamic measurements. These characteristics can serve as a basis for structural damage detection, condition assessment, vibration mitigation, and long-term health monitoring. Over the past few decades, a vast amount of literatures for structural parameter identification with known input and output data can be found in three comprehensive review reports [1–3]. In engineering applications, however, it is difficult or even impossible to measure the excitation of actual structures. Therefore, ambient vibration tests have recently become a more acceptable alternative in parameter identification of civil engineering structures [4]. Example system identification methods with ambient vibration include: peak-picking from power spectral densities [5], natural excitation technique (NExT) [6], and stochastic subspace method [7]. However, most of them faced a challenge in identifying the modal parameters of a structure with closely-spaced modes, particularly in the presence of significant measurement noise from ambient vibration tests.

More recently, time–frequency analysis with Wavelet and Hilbert transforms has received increasing attentions in modal parameter identification of structures [8–11]. Even so, it is still difficult to select appropriate wavelet parameters (e.g. center frequency and bandwidth) for a proper separation of closely-spaced modes. To overcome this difficulty, Teng and Zhu [12] used an adaptive genetic algorithm to optimize the center frequency and bandwidth of a signal. Kijewski-Correa and Kareem [13,14] concluded that continuous wavelet transform can separate two closely-spaced cosine terms using a wavelet function with the appropriate center frequency and bandwidth, and that wavelet transform is advantageous over empirical mode decomposition (EMD) in extracting signals embedded in severe noise.

Hilbert transform is another time–frequency analysis method. It has been directly applied to system identification and damage detection of single-degree-of-freedom systems [15–17]. Limited to single frequency component signals, these applications had

* Corresponding author. Address: 328 Butler-Carlton Hall, Department of Civil, Architectural, and Environmental Engineering, Missouri University of Science and Technology, 1401 N. Pine Street, Rolla, MO 65409-0030, USA. Tel.: +1 573 341 4462.
E-mail address: gchen@mst.edu (G. Chen).

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small amplitude variations in comparison with the change of phases. In order to admit a well-behaved Hilbert transform, Huang et al. [18–20] developed EMD and Hilbert spectral analysis, which is referred to as Hilbert–Huang Transform (HHT). EMD separates a complicated data set into a finite number of intrinsic mode functions (IMFs), each having a well-behaved Hilbert transform. Yang et al. [21–23] successfully applied the HHT method together with the random decrement technique (RDT) to identify the natural frequencies and damping ratios of linear structures. Yang et al. [24] further proposed an EMD-based approach to detect the damage time instants and damage locations by identifying any spike resulted from a sudden change of structural stiffness.

Although HHT is a promising method for the feature extraction of nonlinear and non-stationary signals, it faces several challenges in various applications due to lack of mathematical rigorousness [25]. In particular, the modal interaction of a linear system in frequency domain becomes significant as two frequency components of a signal approach to each other [26]. In this case, the cutoff frequencies used in the sifting process with the required intermittency check were determined from the power spectrum of the measured response time history. However, applying a cutoff frequency in frequency domain may cause a significant fluctuation of the signal in time domain due to brick wall effects. Peng et al. [27] used the wavelet packet transform to decompose a general signal into a set of narrow band signals so that single frequency-component IMFs can be derived. Yang et al. [21] applied band pass filtering on the measured free vibration time histories to identify the modal parameters of multiple-degree-of-freedom systems. Chen and Feng [28] presented a technique to improve the EMD based on the waves' beating phenomena. In order to separate two closely spaced frequency components, Wang [29] proposed a wave group method with EMD, which shifts down the frequency of a real signal to increase the ratio of the two components. However, the frequency downshift method is also empirical; its accuracy depends upon the choice of a temp-signal. More recently, Zheng et al. [30] proposed the singular value decomposition method with band pass filtering as a signal processing technique of HHT to extract the parameters of closely spaced modes. As the spacing of the two frequencies becomes very small, it is difficult to design a filter for a complete separation of the two frequency components. Overall, it is still a challenge to consistently and reliably identify the modal parameters of structures with closely spaced modes using existing methods.

The modal parameters of structures can be identified from ambient vibration in three steps by combining EMD with RDT [26,31,32]. First, a structural response is decomposed into many modal responses using a conventional signal decomposition method such as EMD. Each modal response is then sampled to obtain a large number of short-time responses starting at a common threshold value, whose average response resembles the free vibration of the structure. Finally, the average response (free vibration) is used to identify modal parameters.

More recently, a new signal decomposition theorem with Hilbert transform has been developed by Chen and Wang [25] to accurately separate a finite bandwidth signal into a series of narrow band components. For modal parameter identification, the theorem was referred to as analytical mode decomposition (AMD). AMD has been applied to identify the modal parameters of a three-degrees-of-freedom mechanical system under impulsive loading or free vibration [25].

In this paper, the previous study by Chen and Wang [25] is extended to address the following three main issues: (1) identification of closely-spaced modal parameters from ambient vibration with a new RDT–AMD method, (2) quantification of the frequency resolution and time duration effect of the AMD, and (3) validation of the identified modal parameters with shake table tests. In addition, a new energy error index is introduced to provide a good measure of the identification accuracy, taking into account the effects of both amplitude and phase of structural responses.

2. Proposed methodology for parameter identification from ambient vibration

For modal parameter identification from ambient vibration, a new RDT–AMD method is developed in this study. The AMD and the RDT–AMD methods are briefly described below.

2.1. Brief description on AMD

In the HHT method, a general signal $x(t)$ with multiple components can be empirically decomposed into:

$$x(t) = \sum_{i=1}^{n} x_i(t) + r(t) \tag{1}$$

where each component $x_i(t) = A_i(t)cos(\phi_i(t))$ is an intrinsic mode function of the EMD process [18]; $A_i(t)$ and $\phi_i(t)$ are time-varying amplitude and phase angle, respectively; $r(t)$ is a residual signal, which represents noise or the measurement error. The empirical decomposition is not guaranteed to be successful and lead to a single frequency in each component. In the AMD and Hilbert spectral method [25], a time series is decomposed into many components whose Fourier spectra are non-vanishing over mutually-exclusive frequency ranges as briefly stated for system identification in building structures.

Let $x(t)$ denote a time history response (displacement or acceleration) of a n-story building with n natural frequencies (ω_1, $\omega_2, \ldots, \omega_n$) in Lebesque space $L^2(-\infty, +\infty)$ of the real time variable t. It can be decomposed into n modal responses $x_i^{(d)}(t)$ ($i = 1, 2, \ldots, n$) whose Fourier spectra are equal to $\hat{X}(\omega)$ over n mutually exclusive frequency ranges ($|\omega| < \omega_{b1}$), ($\omega_{b1} < |\omega| < \omega_{b2}$), …, ($\omega_{b(n-2)} < |\omega| < \omega_{b(n-1)}$), and ($\omega_{b(n-1)} < |\omega|$). That is,

$$x(t) = \sum_{i=1}^{n} x_i^{(d)}(t) \tag{2}$$

Here, $\hat{X}(\omega)$ is the Fourier transform of $x(t)$, ω represents a frequency variable, and $\omega_{bi} \in (\omega_i, \omega_{i+1})$ ($i = 1, 2, \ldots, n-1$) are $n-1$ bisecting frequencies. Each modal response has a narrow bandwidth in the frequency domain and can be determined by the following equation:

$$x_i^{(d)}(t) = s_i(t) - s_{i-1}(t), \ldots, x_n^{(d)}(t) = x(t) - s_{n-1}(t) \tag{3}$$

$$s_i(t) = \sin(\omega_{bi}t)H[x(t)cos(\omega_{bi}t)] - \cos(\omega_{bi}t)H[x(t)\sin(\omega_{bi}t)] \quad (i = 1, 2, \ldots, n-1) \tag{4}$$

where $s_0(t) = 0$ and $H[\cdot]$ represents the Hilbert transform of the function inside the square bracket.

The Hilbert transform is a linear operator that takes a function, e.g. in time domain, and produces a transform within the same domain. Unlike the band pass filtering technique where rectangular windows introduced in frequency domain distort the response in time domain due to brick wall effects, AMD represents an exact separation of the frequency contents of a time series except for possible end effects associated with the finite length of a building response [25]. Each bisecting frequency in Eq. (4) can be selected around the arithmetic average of its two nearby natural frequencies. The response decomposition is insensitive to the selection of the bisecting frequency [25].