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Optimal design of eccentricity for seismic applications

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ABSTRACT

Making either mass or stiffness eccentric mitigates the translational vibration of systems subjected to base excitation. The level of mitigation depends not only on the amount of eccentricity but also on the frequency ratio (the ration of translational frequency to rotational frequency). This paper proposes a systematic approach for finding the values of eccentricity and frequency ratio that lead to the maximum reduction in translational vibrations. First an optimization problem in frequency domain is formulated. The mean square value of response is selected as the performance index. Two types of constraints including limitations on rotations and eccentricity are imposed. Kanai–Tajimi power spectral density function is used to model the ground motion. After formulating the optimization problem two structural models are studied numerically: a single story building model and a multistory building model. It is observed that using the proposed approach the performance index can be reduced by up to 50%. The time history analyses also indicate significant reductions in displacements. Finally a case study is conducted to compare the performance of the proposed strategy with that of other established passive control methods. The results of the case study show that the proposed method can be as effective as other strategies, as far as displacement control is concerned.

1. Introduction

Engaging more modes in the response of structures can be used to reduce its translational dynamic response. One such an approach is to engage torsional modes through engineered mass/stiffness eccentricity. In other words three dimensional effects caused by eccentricity, can be used to reduce translational response by engaging new modes of vibration [11]. The level of reduction depends on the relationship between dominant translational and torsional frequencies of the structure [4]. This relationship is expressed as the ratio of dominant translational frequency to the dominant torsional frequency and it is called *frequency ratio* ($\Gamma_x = \omega_x^2/\omega_\theta^2$) [4].

Furthermore statistical analysis shows that by introducing eccentricity to systems subjected to earthquake records, the average transitional vibration can be reduced by up to 30%. The eccentric structures, in which the first mode of vibration is torsional ($\Gamma_x \geqslant 1$), have shown to have higher level of mitigation in translational response [4].

The main objective of this paper is to develop a systematic approach to maximize reductions in translational vibration through introducing mass/stiffness eccentricity to the system. In other words the problem that is going to be solved herein can be defined as follows: A structure with a specific translational frequency is given. If the structure is subjected to base excitation, what is the eccentricity and frequency ratio for which the maximum reduction in translational displacements can be achieved? This eccentricity or distribution of eccentricity is called *optimal eccentricity*. For the sake of simplicity, throughout this paper, translational displacement is called displacement.

2. Formulation of the optimization problem

2.1. Governing equations of motion

In the time domain, the equations of motion of a system with $3n$ degrees of freedom subjected to base excitation can be described as:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{C}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{f}_g(t) \tag{1}$$

where $u(t)$ is the $3n \times 1$ displacement vector at the center of the mass, and **M**, **C** and **K** are mass, damping and stiffness matrixes respectively. $\boldsymbol{f}_g(t)$ is the ground motion function and is described as:

$$\boldsymbol{f}_g(t) = -\boldsymbol{M}\boldsymbol{r}\ddot{\boldsymbol{u}}_g(t) \tag{2}$$

In which $\ddot{\boldsymbol{u}}_g(t)$ is the ground acceleration and $\boldsymbol{r}$ is a $3n \times 1$ location vector. It is assumed that earthquake is applied in x-direction only, therefore $\boldsymbol{r}$ is expressed as:

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$$\boldsymbol{r} = [100100100\ldots100]^T \quad (3)$$

Taking the Fourier Transform of both sides of Eq. (1) the analysis could be transformed into frequency domain. The governing equations in frequency domain are:

$$(-\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K})\boldsymbol{U}(\omega) = \boldsymbol{F}_g(\omega) \quad (4)$$

where $i = \sqrt{-1}$ and $\boldsymbol{U}(\omega)$ is the Fourier transforms of $\boldsymbol{u}(t)$. $\boldsymbol{F_g}(\omega)$ is the Fourier transform of ground motion function and is expressed as:

$$\boldsymbol{F}_g(\omega) = -\boldsymbol{MrU}_g(\omega) \quad (5)$$

in which $\boldsymbol{U_g}(\boldsymbol{\omega})$ is the Fourier transform of ground acceleration function.

Assuming:

$$\boldsymbol{Z}(\boldsymbol{\omega}) = -\omega^2\boldsymbol{M} + i\omega\boldsymbol{C} + \boldsymbol{K} \quad (6)$$

then $\boldsymbol{U}(\omega)$ can be readily found from Eq. (4):

$$\boldsymbol{U}(\omega) = \boldsymbol{Z}^{-1}(\omega)\boldsymbol{F}_g(\omega) \quad (7)$$

Usually $\boldsymbol{Z}^{-1}(\omega)$ is shown as $\boldsymbol{H}(\omega)$ and is called frequency response matrix or transfer function. Therefore, in its simplest form, the equation of motion in the frequency domain can be expressed as:

$$\boldsymbol{U}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{F}_g(\omega) \quad (8)$$

If ground motion function $\boldsymbol{f}_g(t)$ is stochastic then the response $\boldsymbol{u}(t)$ would be stochastic as well. In this case the relationship between the Power Spectral Density Function (PSDF) of response and excitation is described as follows:

$$\boldsymbol{S}_u = \boldsymbol{H}(\omega)\boldsymbol{S}_f\boldsymbol{H}^T(\omega) \quad (9)$$

in which:

$$\boldsymbol{S}_f = (-\boldsymbol{Mr})\boldsymbol{S}_g(-\boldsymbol{Mr})^T \quad (10)$$

$\boldsymbol{S_u}$ and $\boldsymbol{S_f}$ are respectively the PSDF matrixes of response and ground motion. Superscript T denotes the transpose or complex conjugate gradient of a matrix or a vector. $\boldsymbol{S_g}$ is the PSDF of the earthquake excitation.

Finally by substituting Eq. (10) in (9) the PSDF of response can be re-written as:

$$\boldsymbol{S}_u = \boldsymbol{H}(\omega)(\boldsymbol{Mr})\boldsymbol{S}_g(\boldsymbol{Mr})^T\boldsymbol{H}^T(\omega) \quad (11)$$

If one is interested in an r-component response vector, it can be found in time and frequency domains respectively by:

$$\boldsymbol{v}(t) = \boldsymbol{B}\cdot\boldsymbol{u}(t) \quad (12)$$

$$\boldsymbol{V}(\omega) = \boldsymbol{B}\cdot\boldsymbol{U}(\omega) \quad (13)$$

where $\boldsymbol{B}$ is a $r \times 3n$ coefficient matrix, the $r \times r$ spectral density matrix for vector $\boldsymbol{u}(t)$ is then given by:

$$\boldsymbol{S}_v = \boldsymbol{BS}_u\boldsymbol{B}^T \quad (14)$$

Eqs. (12)–(14) are particularly useful when only the displacements and/or rotation of top floor are selected for study.

2.2. Power spectral density of ground motion

A major challenge in optimal design for seismic applications is the uncertainties of ground motion. There are different ways to tackle this challenge. A popular method is to use the power spectral density function proposed by Kanai [7], and Tajimi [13] to model the ground motion as a stationary stochastic process. This PSDF is expressed as:

$$S_g(\omega) = \frac{\omega_g^4 + 4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2\omega_g^2\omega^2} \quad (15)$$

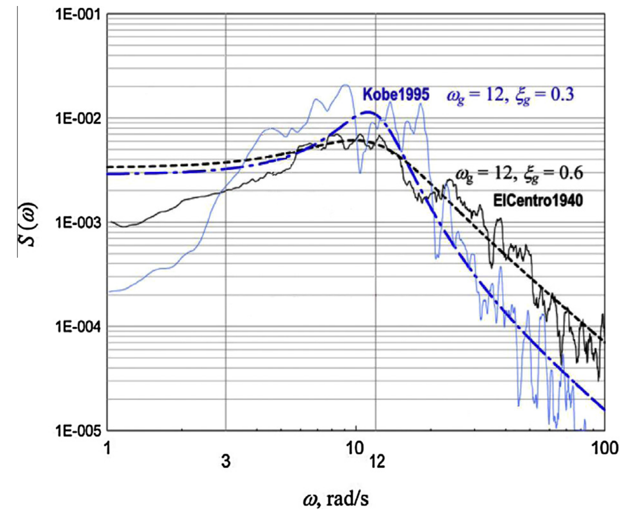

Fig. 1. Comparison of Kanai–Tajimi PSDF with the actual ones for El Centro and Kobe records (Hoang, 2008).

where ω_g and ξ_g are characteristics ground frequency and damping ratio, respectively. By proper selection of these two parameters the above equation can be used to generate different spectral density shapes. It is shown in Fig. 1 that Eq. (15) captures the frequency content of historical seismic events such as El Centro ($\omega_g = 12$ and $\xi_g = 0.6$) and Kobe ($\omega_g = 12$ and $\xi_g = 0.3$) very well [5]. El Centro is the N–S component recorded at the Imperial Valley in El Centro during the Imperial Valley, California earthquake of May 18, 1940 and Kobe is the N–S component recorded at the Kobe Japanese Meteorological Agency (JMA) station during the Hyogo-ken Nabu earthquake of January 17, 1995.

This approach of modeling the ground motion has been widely used in the literature and it has proven to be successful in optimal design of other passive control systems such as tuned mass dampers (TMD) [5,2,9,10].

2.3. Performance index

Since the analyses are performed in the frequency domain and the ground motion is modeled as a stochastic process, a convenient performance index (i.e. objective function) to use is the integral of the PSDF of the structural response with respect to frequency [5,9,6,2]. This integral is basically the mean square value of the structural response. In this research the goal is to minimize the translational vibration of the top floor, therefore the performance function is defined as:

$$J_{top} = E\left(u_{x_{top}}^2\right) = \boldsymbol{l}^T\left(\int_{-\infty}^{+\infty}\boldsymbol{S}_u dw\right)\boldsymbol{l} \quad (16)$$

in which $E(\cdot)$ represents the expected value or mean and $\boldsymbol{l}$ is a $3n \times 1$ vector and is defined as:

$$\boldsymbol{l} = [000\ldots\ldots100]^T \quad (17)$$

2.4. Constraints

Two types of constraints are considered in the problem of finding the optimal eccentricity. One type represents the limitations on the rotation and the other type restrains the maximum value that eccentricity can have.