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An evidence-theory model considering dependence among parameters and its application in structural reliability analysis

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ABSTRACT

Due to its strong ability to deal with the epistemic uncertainty, *evidence theory has been widely used to solve* complex engineering problems with very limited information. This paper proposes a novel evidence-theory model for multidimensional problems, with consideration of the dependence among evidence variables. More specifically, a joint *frame of discernment* (FD) is created for multidimensional problem through an *ellipsoidal model*, where the correlativity between the parameters is well reflected by the shape of the ellipsoid. An approach to construct reasonable ellipsoids using the experimental samples is also provided. Combination of the ellipsoidal model and the marginal *basic probability assignments* (BPAs) of the parameters then generates a joint BPA structure. Based on the new evidence-theory model, a reliability analysis method is formulated to evaluate the safety degree of structures with dependent epistemic uncertainty. Four numerical examples are investigated to demonstrate the applicability of the proposed method.

1. Introduction

Uncertainties associated with material properties, boundary conditions, and loads, etc. exist in numerous practical engineering problems. Quantification of the uncertainties existing in a structure or system and the corresponding reliability analysis have become an essential task for current product design processes. Generally, uncertainties in a system can be classified into aleatory and epistemic types [1]. *Aleatory uncertainty* describes the inherent variability associated with a physical system or environment, which is often treated by the well established *probability model*. *Epistemic uncertainty*, also known as subjective or reducible uncertainty, stems from lack of knowledge or data. This type of uncertainty is reducible in the sense that the uncertainty level will be decreased when more information is collected. Compared with aleatory uncertainty, epistemic uncertainty is much more difficult to deal with. So far, several non-probabilistic models have been developed for epistemic uncertainty, which include evidence theory [2–6], possibility theory [7–9], fuzzy sets [10,11], and convex models [12–21]. For examples, possibility theory deals with the epistemic uncertainty with no conflicting evidence among experts [7–9]. In fuzzy sets theory, a membership function is employed to model the imprecision of an uncertain variable [10,11]. Convex models are developed for cases where only the variation bounds of the uncertainty are available [12–21]. As a generalization of all the above probability and non-probability uncertainty models, evidence theory combines aleatory and epistemic uncertainties in a very straightforward way [22], and is capable of dealing with limited or even conflicting information. Under some special conditions, it will degenerate to the models of possibility theory, fuzzy sets, convex models, etc.

In recent years, evidence theory (also named Dempster–Shafer theory) has obtained more and more attention in structural reliability analysis due to its strong ability in uncertainty characterization. A lot of exploratory work in this filed has been reported in theoretical and application aspects. For example, Helton et al. proposed a sampling approach for sensitivity analysis of the evidence-theory-based reliability [32]. Mourelatos and Zhou constructed an efficient algorithm for evidence-based design optimization (EBDO) [33]. Guo et al. formulated a RBDO method by combining evidence theory with interval analysis [35]. Du proposed a new reliability analysis model for problems with epistemic and aleatory mixed uncertainties [34], and subsequently developed an efficient sensitivity analysis technique based on this model [22]. Agarwal et al. proposed an evidence-theory-based multidisciplinary optimization algorithm for structural reliability design through a trust region strategy [26]. Tonon et al. used evidence theory to quantify the uncertainty in rock engineering and carried out a reliability-based design of tunnels [23]. Jiang et al. proposed an efficient

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structural reliability method using evidence theory by introducing a non-probabilistic reliability index approach [36]. Soundappan et al. compared evidence theory and Bayesian theory in the aspects of uncertainty modeling and decision making [25]. Oberkampf and Helto summarized the strengths and weaknesses of evidence theory in reliability analysis through a simple algebraic function [24]. Alyanak et al. developed a reliability-based design optimization (RBDO) for structures with evidence variables by using a gradient projection technique [31]. Helton et al. gave a unified framework for uncertainty propagation analysis of structures for some different models such as probability model, evidence theory, possibility theory and interval analysis [27]. Bae et al. proposed an efficient reliability analysis method for evidence theory through creating a multi-point approximation for the limit-state surface [28–30], which was an important contribution for efficiency improvement in this area.

In the above mentioned work, the evidence variables are assumed to be independent, i.e., an uncertain variable will not influence other parameters. This implies that those evidence-theory-based reliability analysis models are only applicable to the problems with independent uncertain parameters. However, parameter dependencies exist intrinsically in most systems [39], and have profound implications in the results of risk assessment, or even dominate the system performance [37–39], just as the nonlinearity in physical systems [37]. It is therefore essential to develop an evince theory model with consideration of parameter dependences and corresponding methods in uncertainty modeling and reliability analysis.

This paper aims to propose a new evidence-theory model to quantify the uncertainty with parameter dependence, and furthermore develop a reliability analysis method based on this model. The remainder of this paper is organized as follows. A brief introduction of evidence theory is given in Section 2. The new evidence-theory model considering the dependence of parameters is proposed in Section 3. A reliability method based on the proposed evidence-theory model is given in Section 4. Four numerical examples are investigated in Section 5 and concluding remarks are drawn in Section 6.

2. A brief introduction of evidence theory

Evidence theory was proposed by Shafer [2] and its main concept is that our knowledge on a specific problem can be inherently imprecise. In evidence theory, the bound result which consists of both belief and plausibility is employed to describe the uncertainty or reliability of a problem.

2.1. Frame of discernment (FD)

For a problem with uncertainty, the likelihood of the occurred events will generally take some possible sets that might be nested in one another or partially overlap. The frame of discernment (FD) is defined by the finest possible subdivisions of the sets, and the finest possible subdivision is called the elementary proposition. FD is similar as the finite sample space of a random variable in probability theory, and it is formed by all finite elementary propositions. For example [29], if FD is given as $X = \{x_1, x_2\}$, we have two mutually exclusive elementary propositions x_1 and x_2. All the possible subset propositions of X will form a power set 2^X, and for the above example it has $2^X = \{\emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$. In this paper, we use X to denote either an evidence variable or an FD.

2.2. Basic probability assignment (BPA)

As an important concept in evidence theory, the basic probability assignment (BPA) describes the degree of belief in a proposition. The BPA is assigned through a mapping function m: $2^X \rightarrow [0, 1]$, which satisfies the following three axioms:

$$m(A) \geqslant 0 \quad \text{for any } A \in 2^X \tag{1}$$

$$m(\emptyset) = 0 \tag{2}$$

$$\sum_{A \in 2^X} m(A) = 1. \tag{3}$$

For a set $A \in 2^X$, the BPA $m(A)$ can be interpreted either as the degree of evidence supporting the claim that a specific element of X belongs to A or as the degree to which we believe that such a claim is warranted [33]. A set A with $m(A) > 0$ is referred to as a *focal element* of m.

2.3. Combining rule of evidence

Evidence obtained from independent sources or experts should be combined. For two BPAs $m_1(B)$ and $m_2(C)$, the combined evidence m can be calculated by the Dempster's rule of combining [2]:

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} \quad \text{for } A \neq 0 \tag{4}$$

where

$$K = \sum_{B \cap C = 0} m_1(B) m_2(C) \tag{5}$$

where K represents the total conflict between the two sources or experts. Dempster's rule filters out any conflict, or contradiction among the provided evidence, and hence it is usually suitable for evidence with relatively small amounts of conflict.

2.4. Belief and plausibility measures

Due to the limited information about the uncertainty, it is more reasonable to present the total degree of belief through a bound measure, instead of a deterministic value like in probability theory. The total degree of belief in a proposition A is then expressed by an interval [Bel(A), Pl(A)], where the two bounds, namely the belief measure $0 \leqslant \text{Bel}(A) \leqslant 1$ and the plausibility measure $0 \leqslant \text{Pl}(A) \leqslant 1$, are given as:

$$\text{Bel}(A) = \sum_{C \subseteq A} m(C) \tag{6}$$

$$\text{Pl}(A) = \sum_{C \cap A \neq \emptyset} m(C) \tag{7}$$

As a measure of belief, Bel(A) is obtained by summation of all the BPAs of propositions that totally agree with the proposition A. In contrast, Pl(A) is the sum of all the BPAs of propositions that is not conflicting with the proposition A (a measure of plausibility). These two measures then constitute an interval with lower and upper probability bounds. Bel(A) and Pl(A) have the following properties:

$$\text{Bel}(A_1 \cup A_2) \geqslant \text{Bel}(A_1) + \text{Bel}(A_2) - \text{Bel}(A_1 \cap A_2) \tag{8}$$

$$\text{Pl}(A_1 \cap A_2) \leqslant \text{Pl}(A_1) + \text{Pl}(A_2) - \text{Pl}(A_1 \cup A_2) \tag{9}$$

3. A new evidence-theory model considering dependence

For a multidimensional problem in traditional probability analysis, a joint probability distribution is generally constructed from the marginal probability distributions of all the random variables and their correlation coefficients. Similarly, we need to formulate