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Distortion of thin-walled beams of open section assembled of three plates

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ABSTRACT

The cross-section distortion of thin-walled beams of open section assembled of three plates with two axis of symmetry is considered analytically. The consideration is based on the assumption that stresses and displacements due to the cross-section distortion have small influence on stresses and displacements according to the classical theories of thin-walled beams of open sections, where the cross-section contour maintains its shape, and may be ignored. It is assumed that stresses and displacements due to the cross-section distortion are proportional to the angle of torsion due to shear, according to the theory of torsion of thin-walled beams of open cross-sections with influence of shear. The results are compared with the finite element method, by using shell elements. Several examples are examined by both methods.

1. Introduction

Thin-walled beams are appropriate for designing contemporary ship, aircraft, car and building structures [1–6]. They are characterized by an extremely favorable ratio of flexural to torsional stiffness as well as by the ratio of carrying capacity to the quantity of built-in material. Whereas in building structures thin-walled beams have mainly been analyzed as independent carrying elements, ship, aircraft and car structures are idealized by means of thin-walled beam systems mostly with open or closed-open cross-sections.

The basic assumption in the analysis of thin-walled beams is that the cross-section do not deform in its plane [1].

Takahashi was among the first who investigated the distortion of beams of open cross-section with at least one axis of symmetry [7,8].

Jönnson defined the unique distortion sectorial coordinate for open and closed cross-section and generalized the classical theory of thin-walled beams of open or closed cross-section by including distortional mode of deformation [9,10].

In papers published so far, the cross-section distortion has been related to deformations that occur in beams assembled of four or more component plates. Beams assembled of only three plates have not been taken into consideration.

Another kind of distortion occurs due to bending of individual plates [7], or due to shear deformation of diaphragms at the joint of open and closed sections [11], and is considered as a localized phenomenon.

In recent investigations of the buckling behavior of thin-walled beams, a number of authors studied the effect of lateral distortional buckling [12–14].

However, the cross-section distortion introduced in this paper can be analyzed independently.

In this paper, the influence of distortion of the cross-section during torsion of thin-walled beams of open section assembled of three plates is explored, where it is assumed that the torsion load acts in the beam walls. Cross-sections with two symmetry axes are considered. Additional stresses and displacements occurring due to the cross-section distortion are investigated in relation to those stresses and displacements given by the classical theory of torsion of thin-walled beams, where it is assumed that cross-sections maintain their shapes.

Both distributed moments of torsion per unit length and concentrated moments of torsion are considered, whereby external loads are distributed in the beam walls. The influence of the deplanation of cross-section due to shear (torsion with the influence of shear) is included [4,15–17].

The obtained analytical expressions for stresses and displacements explored in this paper can be brought in line with distortion occurring at bending of thin-walled beams of closed or open

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cross-section [18,20,21], as well as with the recent investigation on distortion occurring at torsion of thin-walled beams assembled of three plates [22].

The obtained analytical solution for stresses and displacements – that include expressions proposed by the classical theory of torsion which assumes unchanged shape of the cross-section during deformation together with the additional expressions that take into account stresses and displacements due to the change of the shape of cross-section – will be proved by means of the finite element method.

The classical theory of torsion of thin-walled beam [1] as well as the theory of torsion with influence of shear [4,15,16], where it is assumed that the shape of cross-section does not change, will be complemented with expressions that take the effect of the cross-section distortion into consideration. Obtained expressions will be presented in relation to expressions of the classical torsion theory.

Derived expressions are appropriate for the analysis in a parametric form, especially in the early stages of the structure design.

2. General equations

2.1. Assumptions

1. The cross-section distortion has a small influence on displacements and stresses according to classical theories of torsion of thin-walled beams of open cross-section, where the cross-section maintain its shape during deformation [1–5], that may be ignored.
2. Displacements and stresses related to the cross-section distortion are proportional to the angle of torsion due to shear according to the theory of torsion of thin-walled beams of open cross-section with influence of shear [4,15].
3. The beams are simply supported at their ends, where it is assumed that the cross-section maintains its shape, i.e. where the cross-section distortion vanishes.

2.2. Constraints

1. The thin-walled beams are assembled of only three plates.
2. The cross-sections are double symmetric.
3. The material is isotropic, linearly elastic.

Ad. 1. Assumption. Beams assembled of three plates are treated with the cross-section distortion that has only a "local character" [7,8], without significant influences on the stresses and displacements according to the classical theories of torsion of thin-walled beams.

Ad. 2. Assumption. If the beam component plates are hinged at their junctions, the displacements due to shear are maximal. The Saint–Venant component of torsion is neglected, while the warping component of torsion is maximal. The cross-section distortion is maximal, defined by the primary angle of distortion $\alpha'_d = \alpha'_d(x)$, and equal to the angle of torsion due to shear $\alpha_s = \alpha_s(x)$,

$$\alpha'_{d,\max} = \alpha_s, \tag{1}$$

where for the simple supported beam [4,15]

$$\alpha_s = \frac{B}{GI_{Ps}}, \tag{2}$$

where $B = B(x)$ is the bimoment, G is the shear modulus;

$$I_{Ps} = \frac{I_\omega^2}{\int_A \left(\frac{S_\omega^*}{t}\right)^2 \mathrm{d}A} \tag{3}$$

is the shear polar second moment of area [4,15];

$$I_\omega = \int_A \omega^2 \mathrm{d}A, \quad S_\omega^* = \int_A \omega \, \mathrm{d}A^* \tag{4}$$

are the sectorial second moment of area and the sectorial moment of the cut-off portion of area, respectively; $\omega = \omega(s)$ is the sectorial coordinate with respect to the principal pole (torsion/shear center) and $A^* = A^*(s^*)$ is the area of the cut-off portion of the cross-section; s is the curvilinear coordinate and s^* is the curvilinear coordinate of the cut-off portion of area, where $\mathrm{d}s^* = -\mathrm{d}s$

Namely, a portion of the beam of unit width can be considered as a hinged frame where the horizontal frame rotates with the angle of torsion due to shear with respect to the principal pole P, while the vertical frame remain vertical (Fig. 1a).

If vertical beams are hinged at their ends and clamped to the horizontal totally rigid beam (Fig. 1b), vertical beams will be subjected to bending by the moment per unit length $M = M(\alpha_s)$,

$$M = F\frac{b}{2}, \tag{5}$$

where $F = F(\alpha_s)$ is the force per unit length due to cross-section distortion and b is the distance between the vertical beam components.

If vertical beam components are considered as rigid beams, the horizontal beam component will be subjected to bending by two opposite moments M, given by (5) (Fig. 1c).

If $\alpha_s = 0$, the cross-section distortion vanishes. The cross-section maintains its shape, as in case of classical theories of thin-walled beams of open cross-section [1–3].

The cross-section distortion may be considered more "exactly" by using the shell theory (by assuming membrane and plate deformations). In that case, due to the cross-section symmetry, the compatibility of vertical linear displacements as well as angular displacements in the cross-section planes at the plate junctions has to be satisfied (Fig. 2a):

$$v_0^* = w_1^*, \quad \varphi_{y0}^* = \varphi_{y1}^*, \quad N_{y0}^* = Q_{y1}^*, \quad M_{y0}^* = M_{y1}^*, \tag{6}$$

where $v_0^* = v_0^*(x)$ and $w_1^* = w_1^*(x)$ are vertical linear displacements of vertical and horizontal plates, respectively, $\varphi_{y0}^* = \varphi_{y0}^*(x)$ and $\varphi_1^* = \varphi_1^*(x)$ are angular displacements of the vertical side and horizontal plates, respectively; $N_{y0}^* = N_{y0}^*(x)$ and $M_{y0}^* = M_{y0}^*(x)$ are the normal force and the bending moment per unit length of vertical plates, respectively, $Q_{y1}^* = Q_{y1}^*(x)$ and $M_{y1}^* = M_{y1}^*(x)$ are the shear force and the bending moment per unit length of the horizontal plate, respectively.

The analytical solution of the problem given by (6) will be preferable since it includes the cross-section distortion in an exact way. However, it becomes obvious that such solution would be rather difficult. Therefore, it would be hardly applicable in the engineering practice, especially in the analysis of complex thin-walled structures.

The vertical plates can also be considered as thin-walled beams of rectangular cross-section subjected to bending with influence of shear in the plate planes under forces per unit length $q = q(x)$ and to pure torsion by moments $M_{x0} = M_{x0}(x)$; the horizontal plate will be subjected to pure torsion only (Fig. 2b). In that case it is assumed the cross-section maintains its shape. Thus, the engineering approach can be applied, as for instance in the theory of torsion of thin-walled beams of open section with influence of shear [4,15]:

$$B = M_{y0}b, \quad M_\omega = Q_{z0}b, \quad M_P = 2M_{x0} + M_{x1} \tag{7}$$

where $M_P = M_P(x)$ is the moment of torsion, $M_\omega = M_\omega(x)$ is the warping moment of torsion and $M_P = M_P(x)$ is the moment of torsion of the beam; $M_{y0} = M_{y0}(x)$ is the bending moment and $Q_{z0} = Q_{z0}(x)$ is the shear force of the vertical components of the beam;