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Analytical modeling of multilayered dynamic sandwich composites embedded with auxetic layers

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ABSTRACT

The influence of viscoelastic and auxetic (Negative Poisson's Ratio NPR) layers on the free vibration modeling of a composite sandwich structure is analyzed in this work. For such purpose, an analytical formulation is performed based on the specification of kinematic theories in terms of displacement fields within each layer of the structure. For instance, simple theories like classical laminate theory (CLT) and high order shear deformation theory (HSDT) are considered. This enables the definition of a dynamic problem for 5-layered sandwich composite using the principle of virtual displacement. Moreover, modal parameters in terms of natural frequencies and associate loss factors are given through the transversal displacement. For linear and isotropic layers, numerical applications highlight the result of the Poisson's ratio on the modal responses. It is particular observed that the natural frequency and loss factor show an increase with respect to the auxeticity of the viscoelastic layers while no significant variations related to the shear function are noticed from one theory to another.

1. Introduction

The idea of the use of auxetic composite materials (materials or structured having an overall negative Poisson's ratio) has been generalized in technological applications since the first works developed by Lakes [1] in this fields. Auxeticity has now been identified in a wide range of man-made materials, including foams, liquid crystalline polymers and micro/nano-structured polymers [2–5]. Due to some interesting properties related to viscoelastic, absorption and the deformation behaviors [6,7], auxetics tend to be used for improving the macroscopic response of structures (see [8,9] for more).

Nowadays, the vibration suppression in mechanical structures is a problem in engineering science that has occupied researchers for a while [10]. The viscoelastic damped structures consist on a sandwich composite in which at least one viscoelastic layer is sandwiched between elastic ones. Also, is it well established that an efficient way to increase passive damping is the use of sandwich construction with alternating elastic and viscoelastic layers [10]. For these structures, the overall damping parameters such as the natural frequency and loss factor are requested in the dynamic analysis since pioneering work, on sandwich beams consisting of isotropic face plates and a viscoelastic core, done by Ross et al. [11]. Exhaustive review dealing with dynamic analysis of sandwich composites is presented in [12]. The solutions of such requests pass upon adequate kinematic models, not only to obtain a reasonable computational cost, but also a proper account of the shear in the viscoelastic layers [13]. Also, the damping is highly related to the transverse shear in the viscoelastic layers. Indeed, Rao et al. [12] emphasis that the dominant mechanism of damping is the shear induced between the damping layers and the constraining layers. Then, one of the ways to take advantage of high damping is the use of an increased or high transverse shear modulus. For instance, isotropic materials show high shear modulus with the auxeticity. That is illustrated by $(1+\nu)^{-1}$ dependency of this property which assumes its increase when the Poisson's ratio tends towards -1.

In the case of two dimensional modeling of sandwich composites, several theories propose different kinds of kinematics for plates, beams and shells. So, to achieve a continuous through-the-thickness distribution of the transverse shear with zeros at the sandwich faces, models like classical laminate theories (CLT), first order-shear deformation theories (FSDTs) or high-order shear deformation theories (HSDTs) can be derived in the consideration of the displacement expansion up to linear, quadratic or high powers, polynomial or sinus expansion of the thickness coordinate. A good survey of the above theories can be found in [10,13–15].

The main objective of this paper is to take advantage of auxetic viscoelastic layers in the macroscopic response of a sandwich composite through a free vibration analysis. The analysis deals

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essentially with the dynamic response of (5)-layered sandwich composite made of auxetic viscoelastic layers sandwiched between elastic ones, by the means of an analytical kinematic formulation. To this end, a simple and available kinematics is assumed within each layer of the structure and enables by the application of the virtual displacement principle the derivation of the modal responses in terms of natural frequency and loss factor.

2. Kinematic formulation

The topology of the structure (Fig. 1) is composed of five (5) layers with two auxetic viscoelastic layers sandwiched between three elastic ones whereas the structure is assumed to be two dimensional model. Let us denote x and z the longitudinal and the transverse coordinate axis. The thickness of the elastic and viscoelastic layers are denoted H_e and H_v, respectively. The length of the beam and its total thickness are L and H_t, respectively. The displacement field within each layer is governing by:

$$U(x,z,t)=\begin{cases} u(x,z,t)=u_0(x,t)-z\frac{\partial w(x,t)}{\partial x}+f(z)\beta(x,t), \\ w(x,z,t)=w(x,t), \end{cases} \tag{1}$$

where $f(z)$ and $\beta(x,t)$ denote the shear function and the additional rotation of the normal to the mid-plane, respectively. $u_0(x,t)$ represents the longitudinal displacement of the mid-plane whereas $w(x,t)$ is the transverse displacement that remains constant in the thickness direction. According to the value of the shear function $f(z)$, several models can be derived [13]:

- Model-1: $f(z)=0$, CLT based model;
- Model-2: $f(z)=z$, Mindlin–Timoshenko model;
- Model-3: $f(z)=z\left(1-\frac{4z^2}{3H_t^2}\right)$, HSDT based on Reddy kinematic's;
- Model-4: $f(z)=\frac{H_t}{\pi}\sin\left(\frac{\pi z}{H_t}\right)$, HSDT based on Touratier kinematic's.

3. Governing equations of the sandwich beam

3.1. Balance of momentum

The dynamic response of the structure is determined using the principle of virtual displacement such as:

$$P_{acc}(\delta u)=P_{\text{int}}(\delta u)+P_{ext}(\delta u). \tag{2}$$

where $P_{acc}(\delta u)$, $P_{\text{int}}(\delta u)$ and $P_{ext}(\delta u)$ denote inertial, internal and external virtual works, respectively. The dynamic problem is described under the following conditions:

$$\begin{cases} P_{acc}(\delta u)=\int_0^L\left(\sum_{k=1}^{5}\rho_k S_k\right)\frac{\partial^2 w}{\partial t^2}\delta w\mathrm{d}x, \\ P_{ext}(\delta u)=0. \end{cases} \tag{3}$$

The internal virtual work for such problem can be written as:

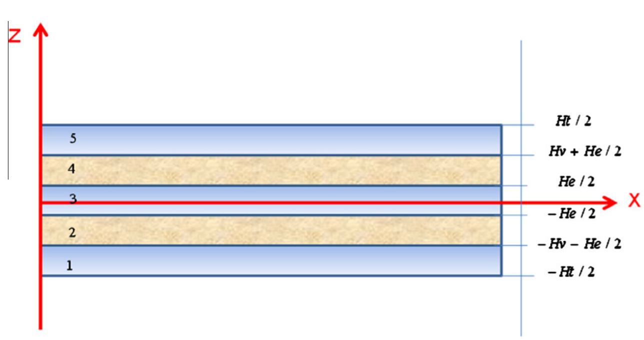

Fig. 1. Topology of the 5-layered sandwich composite.

$$P_{\text{int}}(\delta u)=-\int_{\Omega t}\sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}\Omega t=-\sum_{k=1}^{5}\left(\int_{\Omega k}\sigma_{ij}\delta\varepsilon_{ij}\mathrm{d}\Omega k\right), \tag{4}$$

In Eq. (4) Ωt represents the total volume of the sandwich beam. Assuming a transverse normal stress nil $\sigma_{zz}=0$, Eq. (4) is well written as:

$$P_{\text{int}}(\delta u)=-\sum_{k=1}^{5}\left[\int_{\Omega k}(\sigma_{xx}\delta\varepsilon_{xx}+2\sigma_{xz}\delta\varepsilon_{xz})\mathrm{d}\Omega k\right]. \tag{5}$$

By the means of the above kinematic formulation, the strain field within the layers of the sandwich is defined by:

$$\begin{cases} \varepsilon_{xx}=u_{,x}=-zw_{,xx}+f(z)\beta_{,x}, \\ \varepsilon_{xz}=\frac{1}{2}(u_{,z}+w_{,x})=\frac{1}{2}f_{,z}\beta. \end{cases} \tag{6}$$

Using Eq. (6) in Eq. (5) leads to:

$$P_{\text{int}}(\delta u)=-\sum_{k=1}^{5}\left[\int_{\Omega k}(-\sigma_{xx}z\delta w_{,xx}+\sigma_{xx}f(z)\delta\beta_{,x}+\sigma_{xz}f_{,z}\delta\beta)\mathrm{d}\Omega k\right]. \tag{7}$$

To simplify demonstrations, the new shortened notations are adopted in the remaining part of this paper:

$$\begin{cases} Mt=\int_{St}-z\sigma_{xx}\mathrm{d}St; \\ Ma=\int_{St}f(z)\sigma_{xx}\mathrm{d}St; \\ Q=\int_{St}f_{,z}\sigma_{xz}\mathrm{d}St. \end{cases} \tag{8}$$

where St represents the total surface of the sandwich beam. Therefore, the principle of virtual displacement in Eq. (2) is recasting as:

$$\int_0^L(Mt\delta w_{,xx}+Ma\delta\beta_{,x}+Q\delta\beta)\mathrm{d}x=-\int_0^L\left(\sum_{k=1}^{5}\rho_k S_k\right)\frac{\partial^2 w}{\partial t^2}\delta w\mathrm{d}x. \tag{9}$$

3.2. Solution of the dynamic problem

Let us suppose that the elastic and viscoelastic materials used in this work are linear, homogeneous and isotropic. The Hooke's law for the considered layer yields:

$$\sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\delta_{ij}Tr(\varepsilon), \tag{10}$$

where $\mu=E/2(1+\nu)$ and $\lambda=E\nu/(1+\nu)(1-2\nu)$ with E the Young modulus of the layer and ν its Poisson's ratio.

The solution of the viscoelastic problem related to the associated layers can be derived from the elastic–viscoelastic correspondence principle. Moreover, in the time domain, a Carson–Laplace transform (C–LT) may be applied to transform the time variable t into p; the viscoelastic problem is therefore converted into its associated elastic one. This useful approach has been widely used to tackle viscoelastic problems [16]. For instance, in frequency domain, one deals more with steady-state oscillatory forcing conditions [17]. So, the viscoelastic Young modulus can be written:

$$\begin{cases} E^*(\mathbf{j}\omega)=\widehat{E}(p), \\ p=\mathbf{j}\omega, \end{cases} \tag{11}$$

where ω is the frequency, yielding to the so-called complex modulus that is expressed as:

$$E^*(\mathbf{j}\omega)=E(\omega)[1+\mathbf{j}\eta(\omega)]. \tag{12}$$

In Eq. (12), $E(\omega)$ and $\eta(\omega)$ denote the storage modulus and the loss factor, respectively. The above Hooke's law remains valid for these developments.

The integration by parts of Eq. (9) leads to the basic equation of the dynamic problem: