Engineering Structures 57 (2013) 339–355

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Evaluation of the ultimate strength of R.C. rectangular columns subjected to axial force, bending moment and shear force

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ARTICLE INFO

Article history:
Received 26 March 2013
Revised 27 July 2013
Accepted 2 September 2013
Available online 26 October 2013

Keywords:
Rectangular cross-section
Shear force
Axial force
Bending moment
Reinforced concrete

ABSTRACT

This paper describes a mathematical tool for the calculation of the ultimate strength of reinforced concrete rectangular columns subjected to combined internal forces including shear. The proposed method is based on the application of the static theorem of limit analysis and considers simplified stress fields to simulate stresses in steel bars and (unconfined and confined) concrete. Both truss and arch effects are taken into account. To assess the effectiveness of the method, the relations developed are applied with reference to a large number of columns tested in the past by many researchers and a comparison between the theoretical and experimental results is drawn. Finally, the results of the proposed method are compared to others deriving from the application of more simplified methods present in the literature.

1. Introduction

Over the last century, a significant effort has been made to achieve an accurate evaluation of the response of reinforced concrete columns subjected to combined internal forces including shear. Owing to the different background of researchers, numerous approaches, theories and models have been developed to simulate the most significant aspects of the response of such structural members. The pioneering studies carried out by Ritter and Morsch [1,2] have provided fundamental tools and are still at the basis of some proposals for refined truss models [3–5]. Some new theories have been formulated in the second half of the last century (e.g. equilibrium plasticity truss model, modified compression field theory and softened truss model theory) and have been gradually refined to evaluate the structural response by means of continuum models [6–9]. However, as reported by other researchers [10–12], these theories have not been developed to predict accurately the degradation of the shear strength. Further, some of these advanced theories (e.g. the modified compression field model and softened truss model theory) do not constitute a simple tool for practising engineers. To relieve the computational burden required by continuum models, many researchers have proposed macro-models (e.g.[10–22]) and fibre beam–column elements (e.g. [23–28]). The accuracy and the range of application of these models have been gradually extended to comprise the interaction between axial, flexural and shear deformations and the dynamic response of structures up to their collapse (e.g. [5,10–21]). While some of these models have been developed on the basis of the results of the abovementioned refined theories (particularly the modified compression field theory), some others have been formulated using semi-empirical relations of the force and displacement response parameters which mostly characterise the cyclic behaviour of columns (e.g. maximum shear resisting force and degradation of the shear strength with the flexural ductility demand). The simplified shear strength capacity models adopted in the latter cases (e.g. [29–31]) or reported in codes (e.g. [32–36]) traditionally consider the shear strength as the sum of three resisting contributions: concrete and transverse reinforcement contributions of the truss mechanism and contribution of the arch mechanism. All these shear strength models introduce simplifications of the structural behaviour (e.g. an assigned value for the slope of the normal stresses in the web and simplified relations for the depth of the compression zone of the cross-section) and are adjusted on the basis of results of laboratory tests. Some of these models also consider the degradation of the shear strength because of the inelastic flexural deformation; a detailed discussion regarding the different strategies adopted to reduce the shear strength as a function of the displacement ductility demand is reported in reference [37].

In the last decades, as an alternative to the abovementioned formulations, the shear strength of reinforced concrete members subjected to combined internal forces including shear has also been evaluated by means of simplified continuum models. In these models, stress fields are considered instead of resultants of stresses to simulate the response of steel and concrete [9,38] and the basic

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http://dx.doi.org/10.1016/j.engstruct.2013.09.006

Nomencalture

Greek letters

α	confinement effectiveness factor
α_n	confinement effectiveness factor (longitudinal plane)
α_s	confinement effectiveness factor (transverse plane)
$\Delta\theta$	maximum excursion of θ with respect to the angle θ_I
η	f_{cc}/f_c
θ	angle of inclination of the diagonal stress of the concrete in the web
θ_I	angle of inclination of the first crack
ϕ', ϕ'', ϕ'''	angle of inclination of the strut (arch effect type 1, 2a and 2b)
$\rho_{l,tot}$	reinforcement ratio of the longitudinal bars
ρ_{lw}	reinforcement ratio of the flange longitudinal bars
ρ_{sw}	transverse reinforcement ratio
σ_1, σ_2, σ_3	equivalent normal stress in F_1, F_2, and F_3 (truss action)
σ_{c1}, σ_{c2}, σ_{c3}	normal stress of concrete in the zones F_1, F_2, and F_3 (truss action)
$\sigma'_{c2}, \sigma''_{c2}, \sigma'''_{c2}$	vertical compressive stress in the zone F_2 (arch actions type 1, 2a and 2b)
σ_{eff}	effective lateral compressive stress due to confinement
$\sigma_{eff,y}$	effective lateral compressive stress corresponding to yielding of hoops
σ_I	principal tensile stress developed at first crack on the chord parallel to the neutral axis and passing through the centroid of the uncracked, homogeneous cross section
σ_{l1}, σ_{l2},σ_{l3}	normal stress of the longitudinal reinforcement in the zones F_1, F_2, and F_3 (truss action)
σ''_{l1}	tensile stress in the longitudinal bars of the zone F_1 (arch effect type 2a)
σ'''_{l3}	tensile stress in the longitudinal bars of the zone F_3 (arch effect type 2b)
σ_m	mean compressive stress on the gross concrete cross-section
σ_s	normal stress of the transverse reinforcement caused by transverse deformation of concrete
σ_{s3}	normal stress of the transverse reinforcement in the zone F_3
τ	tangential stress in the zone F_3

Roman letters

A	cross-sectional area of the column
A_1, A_2, A_3	area of the zones F_1, F_2, and F_3
A_{slf}	area of the longitudinal reinforcement of the flange
A_{slw}	area of the longitudinal reinforcement of the web
A_{sw}	area of the transverse reinforcement per layer
b	width of the member cross-section
b_i	distance between consecutive engaged bars
b_o	width of the confined core
c	mechanical cover of the longitudinal reinforcement
c_s	mechanical cover of the hoops
$d_{S'}, d_{S''}, d_{S'''}$	depth of cross-sections S', S'' and S'''
f_c	compressive strength of concrete
f_{c2}	reduced compressive strength of concrete under biaxial state of stress
f_{cc}	compressive strength of confined concrete
f_{ct}	tensile strength of concrete
f_{yl}	yield strength of the longitudinal reinforcement
f_{yw}	yield strength of the hoops
h	depth of the member cross-section
h_1, h_2, h_3	depth of the zones F_1, F_2, and F_3
h_o	depth of the confined core
L_V	shear span of the member
M	bending moment of the cross-section
M', M'', M'''	bending moment (arch actions type 1, 2a and 2b)
M_1, M_2, M_3	contributions of zones F_1, F_2, and F_3 to the bending moment (truss action)
M_{exp}	bending moment of the laboratory test
N	axial force of the cross-section
N', N'', N'''	axial force (arch actions type 1, 2a and 2b)
N_1, N_2, N_3	contributions of zones F_1, F_2 and F_3 to the axial force (truss action)
N''_1, N''_2	contributions of zones F_1 and F_2 to the axial force cause by the arch action type 2a
N_{exp}	axial force of the laboratory test
n_1	total number of longitudinal bars laterally engaged by hoops or cross ties
R_V	V_{exp}/V_{num}
s	spacing of the transverse reinforcement
V	shear force of the cross-section
V', V'', V'''	shear force (arch actions type 1, 2a and 2b)
V_{exp}	maximum shear force recorded during the laboratory test
V_{num}	ultimate shear force predicted by means of the proposed method
y_1, y_2	y-coordinate of the separation lines of the central part F_3
$y_{c,lim}$	$h/2$
$y_{l,\ lim}$	$h/2 - c$
$y_{s,lim}$	$h/2 - c_s$
$y_{G_{c2}}$	y-coordinate of the centroid of the area of concrete in the zone F_2
$y_{G_{c3}}$	y-coordinate of the centroid of the area of concrete in the zone F_3
$y_{G_{l1}}$	y-coordinate of the centroid of the area of the longitudinal reinforcement in the zone F_1

theorems of limit analysis are applied to evaluate the shear capacity of the member. Early applications of this method for structural safety check date back to the late 1970s [39] and regard reinforced concrete beams. Since then, the method has been used by several researchers (e.g. see [40–42]) to obtain simple formulae for the shear strength of beams and to comprehend the role of the geometric and mechanical parameters in the resisting truss mechanism of members subjected to the combined action of forces including shear. In this very context, the aforementioned method has been adopted by Recupero et al. [43–45] to define the ultimate capacity interaction diagrams of reinforced and prestressed concrete beams characterised by rectangular, T or I shaped cross-section and subjected to axial force, shear force and bending moment. More recently, the arch action has also been considered within the same mathematical problem to achieve more accurate estimates of the shear strength of columns. The application of this refined model to circular columns has highlighted the accuracy of the results for members with low to high aspect ratios [46].

This paper extends the latter studies to reinforced concrete rectangular columns subjected to axial force, bending moment and shear force. Like the method proposed for circular columns, the one described here is intended to evaluate the maximum shear force resisted by columns and, therefore, does not consider the shear strength degradation due to the inelastic flexural