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Free vibrations of free spanning offshore pipelines

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ABSTRACT

Fast and accurate methods for determining pipeline eigenfrequencies and associated bending stresses are essential to free span design, and hence of great interest to the pipeline industry. The Rayleigh–Ritz approach has been applied in combination with a displacement field taken as a Fourier sine series to obtain a novel, semi-analytical solution to an initially curved Euler–Bernoulli beam on a partial elastic foundation subject to axial force. The proposed methodology for calculation of free span eigenfrequencies determines static and harmonic response with excellent accuracy, and includes non-linear geometric effects and the pipe stiffening effect caused by static deformation due to gravity.

1. Introduction

This paper provides solutions for the harmonic response of offshore pipelines on uneven seabeds. Due to seabed unevenness and operational loads, an offshore pipeline will not touch down on the seabed uniformly along the length of the pipe. The distance between two pipe touchdown points is called a free span. A free spanning pipeline can be modeled as an Euler–Bernoulli beam on partial elastic foundations, where the elastic foundations are introduced only at the free span shoulders (touch-down zones). Hetenyi [1] solved the equations of motion for a beam on a variable elastic foundation, while Timoshenko [2] studied deflections of a freely supported beam with initial curvature subject to transverse and axial forces. Hobbs [3] applied solutions for simpler boundary conditions and adapted boundary condition coefficients to apply for free span scenarios. Fyrileiv and Mørk [4] used combinations of parametric finite element (FE) analyses and the solutions of Hobbs to determine more accurate semi-empirical approximations for the harmonic response. Their approach constitutes the current formulations in Det Norske Veritas' recommended practice provisions DNV-RP-F105 [5], and it includes significantly more details than previous analytical attempts [6]. In addition to the general refinement, the solutions of Fyrileiv and Mørk [4] also include an approximation of the stiffening effect on the cross-flow fundamental frequency caused by the static deformation from the pipeline weight. Due to buckling or catenary effects, or both, the applicability of the simplified Fyrileiv and Mørk solutions is limited to certain ranges of relative free span lengths and compressive axial forces.

No solution seems to be available in the literature, to the authors' knowledge, of the equations of motion for an initially curved beam on a partial elastic foundation subject to axial force. A novel semi-analytical solution to this boundary value problem is developed in the present paper. The solution is based on the widely applied Rayleigh–Ritz approach and the displacement assumption is taken as a Fourier sine series, comparable to previous work [7–12]. Timoshenko [2] also represented the displacement field by a Fourier sine series when determining the deflection of a freely supported, initially curved beam. The methodology suggested in this paper is easily implemented into a computer program and the computed results are found to be in excellent agreement with the results of FE analyses. Compared to the solutions obtained by the Fyrileiv and Mørk method [4], the accuracy is found to be improved and the range of applicability to be extended. The increase in range of applicability will be demonstrated by comparisons to the semi-empirical formulae of Fyrileiv and Mørk [4].

It will also be demonstrated that the semi-analytical approach is radically faster than equivalent FE analyses, thereby making the presented methodology highly suitable for parametric studies of free spanning pipelines. Free span pipeline engineering typically involves significant amounts of parametric studies due to variations in free span lengths, soil stiffness, span gaps, levels of effective axial force etc. along a pipeline route. Consequently, solutions that are faster than corresponding FE analyses are highly desirable for free span engineering software programs such as

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FatFree [13]. Work by for instance Brubak et al. [7] and others have demonstrated the substantial gains in computational efficiency obtained with semi-analytical solutions based on Rayleigh–Ritz approaches. Such approaches have also found wide applications in engineering practice [11,14].

The solution for the harmonic response of initially curved beams on partial elastic foundations is particularly interesting in design and operations engineering for offshore pipelines, since they are exposed to flow from waves and currents. Due to the boundary layer between the rough pipe surface and the surrounding flow, vortices are formed and shed off at the lee side of the pipe [15,16]. Vortex formation and shedding create periodically oscillating pressure differentials in the in-line and cross-flow directions, sometimes leading to resonance. When resonance occurs, the phenomenon is called vortex induced vibrations (VIV). VIV for free spanning pipelines is an important design aspect. For example, subsea pipelines in the Cook inlet in South Alaska experienced 14 failures due to VIV between 1965 and 1976, and the Ping Hu pipeline in the East China sea failed at two locations during the autumn of 2000 due to VIV [17].

2. Problem definition

Fig. 1 displays a typical free span scenario of a pipeline. In the figure, the span length is L_s and the mid span deflection is denoted δ. The pipe has a dry mass including mass of fluid content per unit length m_d and a submerged weight q. For dynamic response calculations one also has to include the effect of the added mass m_a due to acceleration of the surrounding water. The total effective mass m_e is taken as the sum of m_d and m_a. The regions where the pipe touches down on the seabed are labeled span shoulders, and in the following the length of the shoulders will be denoted $L_{shoulder}$. For the purpose of this paper, the pipe-soil interaction will be modeled according to the recommendations in DNV-RP-F105 [5], adopting linear elastic soil stiffness, given as k_{soil} in Fig. 1, but distinguishing between stiffness coefficients for vertical static, vertical dynamic, lateral dynamic, axial static and axial dynamic displacements. The static and dynamic soil spring stiffnesses are termed K_{VS}, K_V, K_L, K_{AXS} and K_{AX} respectively. The effective axial force concept will be applied for calculation of geometric stiffness effects. The effective axial force S_{eff} for an axially unrestrained pipe is zero while the upper bound value, obtained for a pipe fully restrained axially [18,19], is

$$S_{eff} = H_{eff} - \Delta p_i A_i (1 - 2v) - EA_s \Delta T \alpha \tag{1}$$

Here, H_{eff} is the residual lay tension, Δp_i is the internal pressure difference relative to the internal pressure at the time of laying, A_i is the internal area of the pipe cross-section, A_s is the steel cross-sectional area, E is the Young's modulus, v is the Poisson's ratio of the steel, ΔT is the change in temperature from the time of laying and α is the temperature expansion coefficient.

The in-line direction is perpendicular to the pipe axis and parallel to the flow plane. However, as illustrated in Fig. 2, the direction of the incoming flow is not necessarily perpendicular to the pipe. For a horizontal flow, whether perpendicular to or at an angle with the pipe axis, the in-line direction is horizontal and perpendicular to the pipe axis. The cross-flow direction is perpendicular to both the flow and the pipe axis.

Response frequencies have been assigned two subscripts indicating direction and mode number respectively. For instance $f_{IL,2}$ is the frequency of the second in-line mode and $f_{CF,1}$ is the frequency of the fundamental cross-flow mode. For a perfectly straight pipe, the in-line and cross-flow frequencies become equal if the lateral and vertical dynamic soil stiffnesses are equal. However, the static vertical deflection due to gravity will have a stiffening effect, resulting in an increased cross-flow frequency, particularly for the fundamental mode. Variation in the in-line and cross-flow soil stiffnesses also results in different harmonic response in the two directions. Consequently, the in-line and cross-flow harmonic responses will be treated separately.

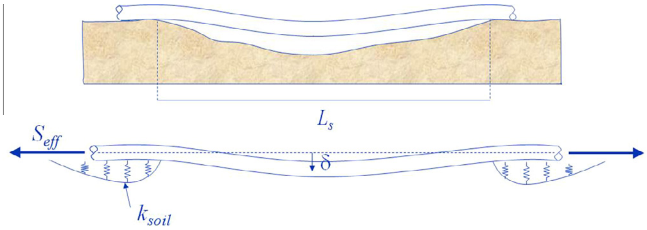

Fig. 1. Free span scenario of a pipeline, indicating free span length L_s and mid-span deflection δ.

The geometry and boundary conditions of the model are idealized as shown in Fig. 3. The shoulders are considered to be horizontal and straight, and the pipeline ends are assumed to be simply supported, allowing no axial or other displacements. This assumption is physically motivated, either by span shoulders that are sufficiently long to provide approximate axial fixity as a result of axial pipe-soil friction, or that other effects from the seabed induces axial fixation (such as neighboring spans, rock dumps, pipe crossings, etc.).

The effective axial forces in the span will be governed by the global behavior of the pipeline, and the free span geometry may be affected by global axial translations ("feed-in") of the pipeline. Details on how to perform global pipeline analyses in order to estimate such effects are given in Det Norske Veritas' recommended practice provisions DNV-RP-F110 [20]. In a local model, the global effects must be estimated and given as input data.

The adopted, simplified boundary conditions (Fig. 3) are conveniently chosen for validation of the semi-analytical model, but they are also in line with the recommendations in DNV-RP-F105 [5], thereby making the comparative studies as relevant as possible. In the numerical computation, the shoulder lengths are taken as three times the span length. The axial friction is disregarded when determining the deflected geometry due to the static loading, but dynamic axial soil stiffness is included in the modal analyses. The lengths of the shoulders are chosen such that the dynamic response is unaffected by them. Long shoulder lengths are necessary to ensure that dynamic axial behavior does not influence the pipe response in cross-flow direction. However, the semi-analytical model presented below is equally applicable for other choices of span shoulder lengths and axial friction behavior.

In the semi-analytical method presented below, three solutions to the problem defined by Fig. 3 will be required:

1. A solution for the static equilibrium case where the pipe is subject to its submerged weight q and effective axial force S_{eff}.
2. A solution to determine the eigenfrequencies, later referred to simply as frequencies, and associated eigenmodes, later referred to as mode shapes, for the linearized harmonic eigenvalue problem in the in-line direction subject to pipe effective mass m_e and effective axial force S_{eff} in the equilibrium configuration.
3. A solution to determine frequencies and mode shapes for the linearized harmonic eigenvalue problem in the cross-flow direction, accounting for the stiffening effect of the vertical static displacement, effective mass m_e and effective axial force S_{eff}.

The static solution is carried out first, in order to establish the static configuration and the corresponding value for S_{eff}. In the