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Investigation of nonlinear restoring stiffness in dynamic analysis of tension leg platforms

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ABSTRACT

Shortcomings of the traditional stiffness matrix in dynamic analysis of TLPs, derived by considering equilibrium of forces, are pointed out, as well as dilemma concerning consistency of the recently presented matrix based on energy balance. New stiffness matrix is derived by utilizing both force equilibrium approach, with algebraic averaging and root mean square of tendon forces, and energy balance approach for large surge and sway. Yaw is treated as a small and large displacement. Static numerical analysis is performed for all six cases by imposing surge force and yaw moment. The obtained results are compared with those of FEM analysis, and useful conclusion is drawn, which can be used for improvement of uncoupled mathematical model of TLPs.

1. Introduction

Tension leg platform (TLP) is classified as a compliant offshore structure, i.e. a semi-submersible one attached to the sea bottom by vertical pretensioned tendons or tethers, [1]. It is ordinary used for deep water oil operations. At the beginning the installed depth was 147 m, *Hutton* (1984), and nowadays it reaches much higher values, for instance 1425 m, *Magnolia* (2005). The consisting parts of a TLP are: pontoon, columns and deck with drilling equipment, [2].

TLP motion in waves is nonlinear due to nonlinear restoring stiffness and damping, [3]. The tendons make TLPs more mobile in horizontal than in vertical plane and ensure almost horizontal position of the working area. Vertical excitation is caused by the first order wave forces, while dominant horizontal excitation is due to the second order wave forces, [4]. Vertical response, i.e. heave, roll and pitch, has high natural frequencies due to high axial tendon stiffness. Natural frequencies of horizontal motion, i.e. surge, sway and yaw, are much lower due to tendon geometric stiffness and can easily fall into resonance with the forcing frequency.

Restoring stiffness plays very important role in TLPs dynamic behavior. Horizontal motion is nonlinear since stiffness is function of surge, sway and yaw. Stiffness of vertical motion is almost linear and depends on platform offset, which causes setdown, as position parameter. Setdown is one of very important design parameters for limiting platform immersion [5].

Nowadays, the secant stiffness matrix introduced in [6] and slightly modified in [7,8] is still widely used for dynamic analysis of TLPs. Its formulation is based on equilibrium of restoring forces due to large displacements. One finite displacement is imposed while the others are restrained. Asymmetric stiffness matrix for six d.o.f. is established with respect to the center of gravity. Shortcomings of that formulation is that stiffness elements of surge, sway and yaw depend on tendon axial stiffness instead of a buoyancy increase due to setdown, as a hydrostatic spring. The former stiffness is much larger than the latter, and since setdown is not taken into account, implication is excessively large stiffness of horizontal motions. By considering equilibrium due to particular displacements, coupling motions is not taken into account. These problems are analysed in details in [9].

Recently, another formulation of nonlinear restoring stiffness for TLPs is presented in [10], specifying also the shortcomings of the above mentioned traditional one. The stiffness matrix is derived by the energy approach and employing Lagrange's equations. Since the tendon setdowns are different due to coupling of surge and sway with yaw, the platform is considered as independent quadrants, which follow the tendon top motion. Potential energy of the system is established under that assumption, and its first derivatives per displacements give the restoring forces, i.e. secant stiffness matrix. The shortcomings of the traditional stiffness are overcome, i.e. stiffness of horizontal motion depends on platform

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hydrostatics, and coupling between surge, sway and yaw is captured.

Dynamic analysis of TLP is performed by the uncoupled and fully coupled models [11]. In the former case platform is considered as a rigid body without tendon influence, and linear restoring is applied. If nonlinear damping is linearized problem can be solved in frequency domain. That advantage of reduction of computing time is paid by decreased accuracy.

Fully coupled model is actually 3D FEM model of platform and tendons adapted to large displacements. Due to mechanical and hydrodynamical nonlinearity problem is analysed in time domain. Nonlinear equation of motion can be linearized that makes some difficulties and limitations.

Motivated by the fact that the uncoupled dynamic analysis of TLPs is widely performed by using linear or an inadequate nonlinear restoring stiffness matrix, research for consistent stiffness is undertaken [9]. The force equilibrium approach is employed and stiffness similar to that in [10], determined by the energy approach, is obtained. New stiffness elements are the same for surge, sway and yaw, but without some additional coupling terms present in the energy formulation [10]. Hence, there is doubt which of the two stiffness formulations is a proper one.

In order to overcome that dilemma nonlinear restoring stiffness is derived in this paper by employing both the force equilibrium and energy balance approach for large surge and sway, as well as yaw motion that was not the case in the previous considerations, [9] and [10]. Then, the obtained stiffness expressions are reduced for the case of small yaw.

In order to evaluate two different restoring stiffness formulations, static response of a TLP platform exposed to surge force and yaw moment is analysed, treating yaw both as a small and large quantity. The obtained results are compared to those determined by FEM analysis that leads to some interesting conclusion.

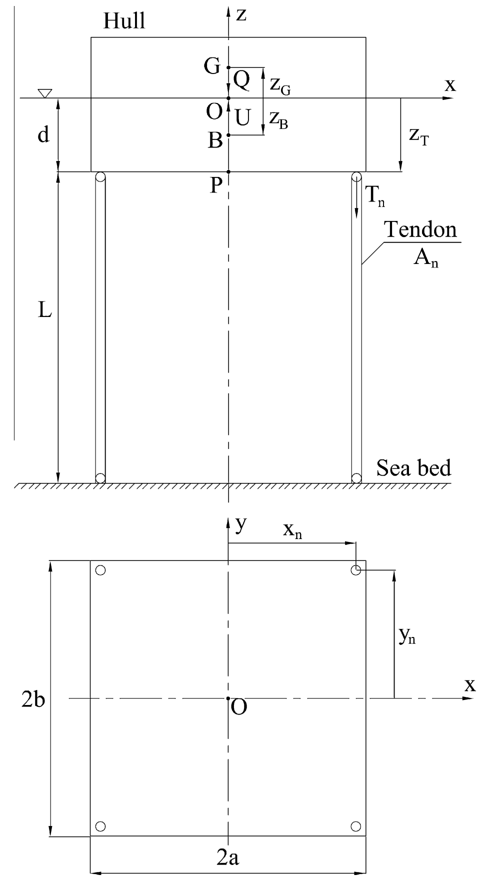

Fig. 1. Double symmetric TLP.

2. Stiffness based on equilibrium of forces

2.1. Large translation and rotation

A double symmetric rectangular TLP, with four tendons and main dimensions shown in Fig. 1, is considered. The platform is exposed to large surge, sway and yaw, δ_x, δ_y and φ, which are common for all tendons, Fig. 2. Trajectory of the tendon top due to yaw is circular arc, $r\varphi$, where r is the tendon radial distance from the platform centroid. The tendon final offset is determined with the secant displacement, Fig. 2

$$\delta_\varphi = 2r\sin\frac{\varphi}{2}. \tag{1}$$

According to Fig. 2 the tendon top coordinates in the local coordinate system read

$$L_x^n = \delta_x - \Delta_x^n = \delta_x - \delta_\varphi \sin\left(\theta_n + \frac{\varphi}{2}\right), \tag{2}$$

$$L_y^n = \delta_y + \Delta_y^n = \delta_y + \delta_\varphi \cos\left(\theta_n + \frac{\varphi}{2}\right), \tag{3}$$

$$L_z^n = \sqrt{L^2 - (L_x^n)^2 - \left(L_y^n\right)^2}$$
$$= L\left\{1 - \frac{1}{L^2}\left(\delta_x^2 + \delta_y^2 + \delta_\varphi^2\right) + \frac{2\delta_\varphi}{L^2}\left[\delta_x \sin\left(\theta_n + \frac{\varphi}{2}\right) - \delta_y \cos\left(\theta_n + \frac{\varphi}{2}\right)\right]\right\}^{\frac{1}{2}}, \tag{4}$$

where L is tendon length and θ_n, n = 1, 2, 3, 4, is tendon central angle.

Components of the tendon tension forces T_n in an offset position are proportional to the tendon top coordinates

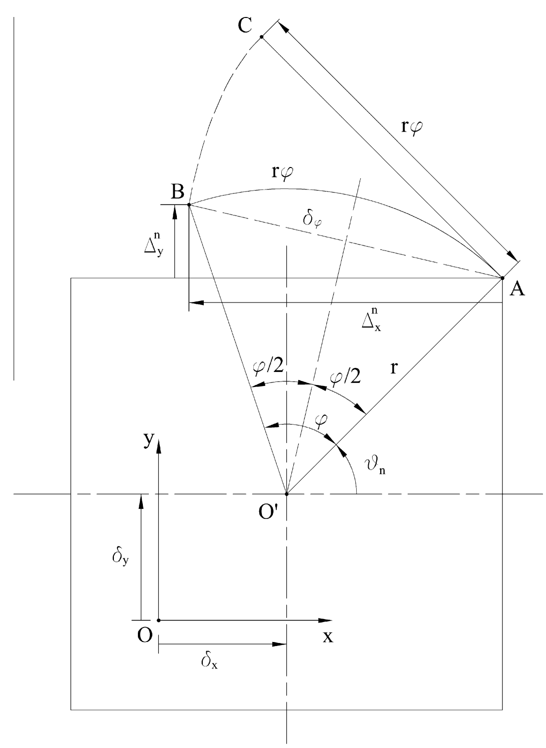

Fig. 2. Large surge, sway and yaw.