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Analysis of flexure-shear behavior of UHPFRC beams based on stress field approach

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ABSTRACT

An adaptation of the Modified Compression Field Theory (MCFT) to grasp the flexure-shear behavior of prestressed or reinforced beams made of Ultra High Performance Fiber-Reinforced Concrete (UHPFRC) is presented. The UHPFRC mechanical behavior is captured using only "stress–strain" relationships obtained from characterization tests and modified with correction factors derived from literature. The model has been checked with respect to complete data of two experimental programs. The predictions of the model agree rather well with the experimental results with an efficient evaluation of the reorientation of the compressive struts with increasing load. The condition for a synergetic contribution to the shear capacity of transversal reinforcement up to their yield strength and UHPFRC is also discussed.

1. Introduction

The Ultra High Performance Fiber-Reinforced Concrete (UHPFRC) behavior under tension is a fundamental constitutive property that modifies the use of conventional reinforcement [1]. In addition the structural applications often associate UHPFRC with pretension all the more that the compressive strength of these materials is quite high. Thus the UHPFRC behavior under tension is often critical, not or not only for bending verification but also for shear design where only UHPFRC carry tensile forces. Thus the behavior of beams made of UHPFRC under shear loading is particularly relevant to illustrate this situation. As an example, shear justification critically determined the web thickness of ITE® beams in Pinel Bridge [2].

For conventional reinforced concrete, at the end of the 19th and at the beginning of the 20th century, Ritter [3] and Mörsch [4] used the equilibrium equations to develop the first truss model in order to determine the ultimate shear strength of reinforced concrete beams. From that time, many researchers have developed truss models capable of predicting not only the ultimate shear strength of reinforced and/or prestressed concrete structures but also their global behavior ("Load–Deflection" curve for example) under shear loading. Several of such models are based on an evaluation of the stress field [5–16]. The Modified Compression Field Theory (MCFT) [5–8] is the most commonly used. In particular, it forms the basis for shear design in the Canadian code [17] and in the Final Draft of *fib* Model Code 2010 [18–20].

In the present paper, the results of a study examining the feasibility of applying an adaptation of the MCFT for the rational assessment of shear behavior in reinforced and prestressed UHPFRC beams are presented. The proposed adaptation of MCFT to UHPFRC has first been checked with respect to complete data of an experimental program realized at IFSTTAR [21–23] and defined to analyse the shear behavior of reinforced and prestressed beams made of UHPFRC taking into account the actual orientation of fibers. Then the model has been verified against data from an experimental campaign carried out by Sato et al. [24] intending in particular to check the additivity of fiber and transverse reinforcement contributions for reinforced UHPFRC beams with or without stirrups.

2. Adopted approach

The approach presented herein is based on MCFT. The fibers participation is integrated in the UHPFRC contribution. The UHPFRC is considered as developing multiple fine-cracking under tension. Thus a "stress–strain" approach can be appropriate, in

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Nomenclature

b_w	web width
h	beam height
d	beam effective depth
x	horizontal coordinate
u, t	refer to horizontal coordinate
y	vertical coordinate
z	internal lever arm
φ	curvature
$v(L/2)$	beam midspan deflection
A_{st}	area of transverse reinforcement
s	spacing between the shear reinforcement (links)
θ	angle of the concrete struts with respect to longitudinal axis
R	resultant of axial force
P_0	prestress force under no load
e_0	strands eccentricity
ε_x	strain // horizontal axis
ε_y	strain // vertical axis
γ_{xy}	shear strain
ε_1	principal tensile strain
ε_2	principal compressive strain
$\varepsilon_{t,\ lim}$	UHPFRC ultimate tensile strain
σ_{sw}	stress inside transversal reinforcement
σ_1	principal tensile stress (absolute value)
σ_2	principal compressive stress (absolute value)
τ	shear stress
$f_{t\text{-}UHPFRC}$	UHPFRC tensile strength
$f_{t\text{-}steel}$	mean yield strength of the passive or active reinforcement
$f_{0.2\%}$	conventional yield strength at 0.2% of the passive or active reinforcement
f_{ult}	maximum strength of the passive or active reinforcement

particular for the UHPFRC tensile behavior [25–29]. Concerning UHPFRC mechanical behavior, the proposed approach needs only "stress–strain" relationships obtained from characterization tests and modified with correction factors determined from literature. Compared to the MCFT, only "average" stresses and strains (with a gauge length including many cracks as shown in Fig. 1) are used. It is not necessary to consider local stresses at a diagonal crack.

In a similar manner to MCFT, equilibrium equations, strains compatibility conditions and "stress–strain" relationships are detailed in the following sections.

2.1. Equilibrium equations

The detailed explanations to obtain equilibrium equations are given in particular by Vecchio and Collins [5] and Collins and Mitchell [6].

From Mohr's circle representation (see Fig. 1), the first equilibrium equation involving "average" stresses is deduced:

$$\sigma_1 + \sigma_2 = \tau \cdot (\tan(\theta) + \cot(\theta)) \tag{1}$$

with

$$\tau = \frac{V}{b_w \cdot z} \tag{2}$$

The diagonal compression and tension in UHPFRC transfer vertical force to eventual transversal reinforcement (see Fig. 2):

$$A_{st} \cdot \sigma_{sw} = (\sigma_2 \cdot \sin^2(\theta) - \sigma_1 \cdot \cos^2(\theta)) \times b_w \cdot s \tag{3}$$

From Eqs. (1)-(3), the following equation is deduced:

$$V = \frac{A_{st} \cdot \sigma_{sw}}{s} \cdot z \cdot \mathrm{cot\,an}(\theta) + \sigma_1 \cdot b_w \cdot z \cdot \mathrm{cot\,an}(\theta) \tag{4}$$

Thus the shear capacity is given by the UHPFRC tensile strength and by eventual passive transversal reinforcement.

2.2. Strains compatibility

As for equilibrium equations, the strains compatibility conditions are detailed in Refs. [5,6].

From Mohr's circle representation (see Fig. 1), the first strains compatibility equation relating "average" strains is deduced:

$$\tan^2(\theta) = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_y - \varepsilon_2} \tag{5}$$

The following strains compatibility condition is obtained thanks to the first strains tensor invariant:

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_x + \varepsilon_y \tag{6}$$

2.3. Stress–strain relationships

2.3.1. UHPFRC mechanical behavior

2.3.1.1. "Stress–strain" relationship under compression. The concrete inside the webs of the beams submitted to a concomitant bending and shear load, is not only under compression in the principal direction Nr 2 but also under tension in the principal direction

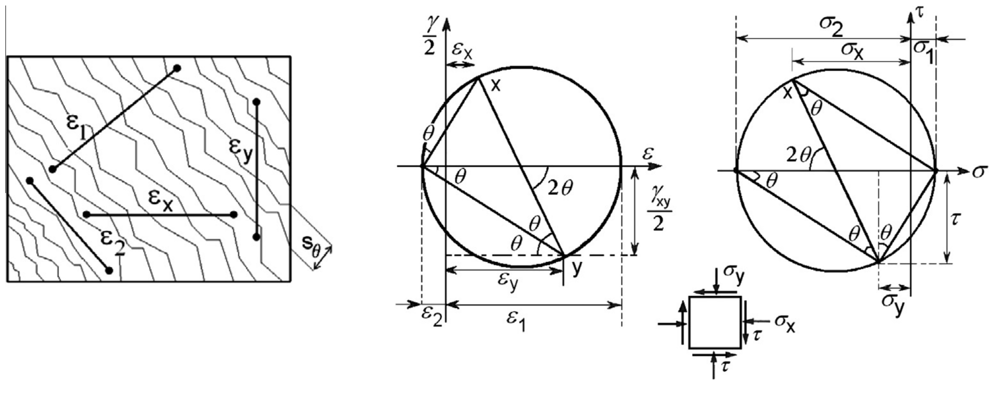

Fig. 1. Notion of "average" strains (at left); Mohr's circle representation of strain and stress field (at right).