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Fibers orientation optimization for concrete beam strengthened with a CFRP bonded plate: A coupled analytical–numerical investigation

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ABSTRACT

Important failure mode of such plated beams is the debonding of the FRP plates from the concrete due to high level of stress concentration in the adhesive at the ends of the FRP plate. This paper presents a new method for reducing interfacial stresses in a concrete beam bonded with the FRP plate by including the effect of the fiber orientation in the FRP plate. This work is divided into two parts; the first one is based on the laminates theory for the analytical solution where a minimization method is used to directly determine the fiber orientation reducing the interfacial stresses. The second part consists into a Finite Element modeling where the analytical solution and different fibers orientation combinations are tested for improving strengthening quality. Numerical results from the present analysis are presented in order to show the advantages of the present solution over existing ones and to reconcile debonding stresses with strengthening quality.

1. Introduction

Strengthening reinforced concrete beams by plating FRP laminates represents a new technology in the civil engineering field. This technique has many advantages including ease of application due to the high strength-to-weight ratio of FRP, conserving aesthetic aspect of the structure, and high corrosion resistance.

One of the main disadvantages of this technique is the debonding of the FRP plates from the concrete, particularly at the ends of the FRP plate. Consequently, many studies have been carried out in order to understand the failure mechanism of plated or connected beams [1–14] or laminated glass [15]. These studies converge to an important aspect: the presence of shear and normal stresses at the plate–core interface. In fact, these stresses can produce the brittle fracture of the concrete layer, which supports the composite laminate, followed by the premature failure of the strengthened beam. Many closed-form solutions have been developed by researchers for the interfacial stresses [16–35]. Smith and Teng's solution [21] gives an accurate estimation of interfacial stresses but does not take into account the FRP plate fiber orientation. Other solutions have been presented in order to improve the solution developed by Smith and Teng [21]. In fact Tounsi [22] have proposed a new approach taking into account the adherends shear deformations and neglecting fibers orientation effect too. Lau et al. [9] have presented a simple theoretical model to estimate the interfacial stresses taking into account the FRP plate fibers orientation. However, this method ignores the plate bending deformations effects and the flexural rigidity of the composite plate is not well estimated to compute the interfacial normal stresses.

Tounsi and Benyoucef [23] have presented a new method in which the FRP plate fiber orientation is considered and the flexural rigidity of the composite plate is not neglected. A sensitivity analysis has been presented considering different fibers orientation combinations.

In this paper, the same approach as Tounsi and Benyoucef [23] for interfacial stresses expressions is considered. However a new method enabling to obtain the minimum of interfacial stresses by minimizing process is presented. This process gives directly the fibers orientation combination presenting the minimum of debonding risk conversely to Tounsi and Benyoucef [23] where a parameter study varying fiber orientation combinations is used. A Finite Element investigation is also presented in this paper to verify the analytical method and to test the strengthening quality of the optimum solution for debonding stresses.

2. System definition and assumptions

The derivation of the solution below is described in terms of adherends 1 and 2 (Figs. 1 and 2), where adherend 1 is the concrete beam and adherend 2 is the soffit plate. Adherend 2 can be either steel or FRP but not limited to these ones.

The following assumptions are made:

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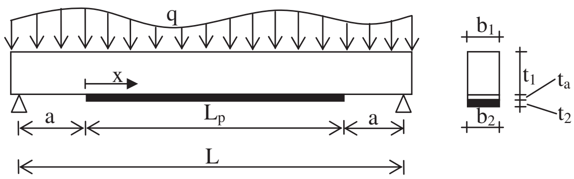

Fig. 1. Simply supported beam strengthened with bonded FRP plate.

1. The concrete, adhesive, and FRP materials behave elastically and linearly.
2. No slip is allowed at the interface of the bond (i.e. there is a perfect bond at the adhesive–concrete interface and at the adhesive–plate interface).
3. Stresses in the adhesive layer do not change with the thickness.
4. Deformations of adherends 1 and 2 are due to bending moments and axial forces.
5. The shear stress analysis assumes that the curvatures in the beam and plate are equal (since this allows the shear stress and peel stress equations to be uncoupled). However, this assumption is not made in the peel stress solution. This assumption is used in several works e.g. Smith and Teng [21] and Tounsi [22].

3. Analytical equations

3.1. Adhesive shear stress: governing differential equation

In this part, q is assumed to be uniformly distributed load.

A differential segment, dx of the plated beam is shown in Fig. 2 where all the forces and stresses are represented with their signs. We denote $\tau(x)$ and $\sigma(x)$, respectively as the interfacial shear and the normal stresses.

The shear strain in the adhesive layer is expressed as:

$$\gamma_{xy} = \frac{\partial u(x,y)}{\partial y} + \frac{\partial w(x,y)}{\partial x} \approx \frac{u_2(x) - u_1(x)}{t_a} \tag{1}$$

Consequently, the shear stress in the adhesive layer is given by:

$$\tau(x) = G_a\left[\frac{u_2(x) - u_1(x)}{t_a}\right] \tag{2}$$

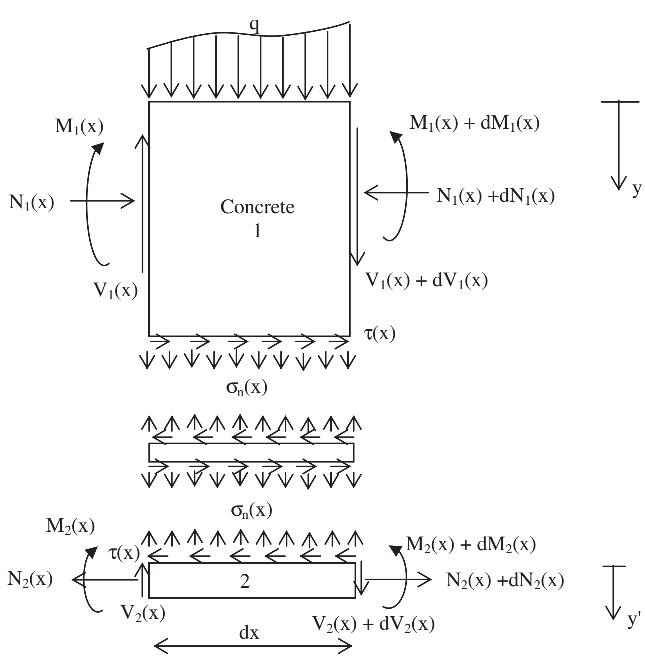

Fig. 2. Forces in the infinitesimal element of a soffit-plated beam.

where G_a, t_a, u_1 and u_2 denote respectively the shear modulus, the thickness of the adhesive layer, the horizontal displacement at the bottom of the concrete beam, and the horizontal displacement at the top of the externally bonded FRP plate. Differentiating Eq. (2) with respect to x gives the shear stress expression in terms of the mechanical strain of the concrete $\varepsilon_1(x)$ and the FRP plate $\varepsilon_2(x)$:

$$\frac{d\tau(x)}{dx} = G_a\left[\frac{\varepsilon_2(x) - \varepsilon_1(x)}{t_a}\right] \tag{3}$$

The strain at the bottom of adherend 1 is given by:

$$\varepsilon_1(x) = \frac{du_1(x)}{dx} = \frac{y_1}{E_1 I_1}M_1(x) - \frac{1}{E_1 A_1}N_1(x) \tag{4}$$

where E_1 is the elastic modulus, A_1 the cross-sectional area, M_1 the bending moment, N_1 the axial force and y_1 the distance from the bottom of adherend 1 to its center.In this study, the laminate theory [36] is used to highlight the fibers orientation effect on the behavior of the externally bonded composite plate. Using this theory for a symmetrical composite plate [36], the mid-plane strain ε_x^0 and the Curvature k_x of the composite plate are given as:

$$\varepsilon_x^0 = A'_{11} N_x \frac{1}{b_2} \text{ and } k_x = D'_{11} M_x \frac{1}{b_2} \tag{5}$$

where $[A'] = [A]^{-1}$ is the inverse of the extensional matrix $[A]$; $[D'] = [D]^{-1}$ is the inverse of the flexural matrix $[D]$; and b_2 is the width of FRP plate.

Explicitly, the terms of the matrices $[A]$ and $[D]$ are written as:

$$A_{mn} = \sum_{j=1}^{N} \overline{Q}_{mn}(h_j - h_{j-1}) \text{ and } D_{mn} = \sum_{j=1}^{N} \overline{Q}_{mn}\left(h_j^3 - h_{j-1}^3\right) \tag{6}$$

where

$$\begin{cases} \overline{Q}_{11} = \left[\frac{E_{11}}{1-\nu_{12}\nu_{21}}\right]\cos^4(\theta_j) + \left[\frac{E_{22}}{1-\nu_{12}\nu_{21}}\right]\sin^4(\theta_j) + 2\left[\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} + 2G_{12}\right]\cos^2(\theta_j)\sin^2(\theta_j) \\ \overline{Q}_{22} = \left[\frac{E_{11}}{1-\nu_{12}\nu_{21}}\right]\sin^4(\theta_j) + \left[\frac{E_{22}}{1-\nu_{12}\nu_{21}}\right]\cos^4(\theta_j) + 2\left[\frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}} + 2G_{12}\right]\cos^2(\theta_j)\sin^2(\theta_j) \\ \overline{Q}_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}\left[\cos^4(\theta_j) + \sin^4(\theta_j)\right] + \left[\frac{E_{11}}{1-\nu_{12}\nu_{21}} + \frac{E_{22}}{1-\nu_{12}\nu_{21}} - 4G_{12}\right]\cos^2(\theta_j)\sin^2(\theta_j) \\ \overline{Q}_{33} = G_{12} \end{cases} \tag{7}$$

where j is number of the layer; h; $[\overline{Q}]$ and θ_j are respectively the thickness, the Hooke's elastic tensor and the fibers orientation of each layer.

Using classical laminate theory, the strain at the top of adherend 2 is given by:

$$\varepsilon_2(x) = \frac{du_2(x)}{dx} = \varepsilon_x^0 - \frac{t_2}{2} \cdot k_x \tag{8}$$

Substituting Eq. (5) in (8) gives the following equation:

$$\varepsilon_2(x) = A'_{11}\frac{N_2(x)}{b_2} - D'_{11}\frac{t_2}{2b_2}M_2(x) \tag{9}$$

where $N_2(x) = N_x$ and $M_2(x) = M_x$.

The subscripts 1 and 2 denote respectively adherends 1 and 2. $M(x)$, $N(x)$ are the bending and axial force in each adherend.

The horizontal forces equilibrium gives:

$$\frac{dN_1(x)}{dx} = \frac{dN_2(x)}{dx} = b_2\tau(x) \tag{10}$$

And then:

$$N_1(x) = N_2(x) = b_2\int_0^x \tau(x)dx \tag{11}$$

Assuming equal curvature in the beam and the FRP plate (perfect contact) it is obtained:

$$\frac{d^2w_2(x)}{dx^2} = \frac{d^2w_1(x)}{dx^2} \tag{12}$$