Engineering Structures 56 (2013) 477–488

Contents lists available at SciVerse ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Peak loads and failure modes of steel-reinforced concrete beams: Predictions by limit analysis

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ARTICLE INFO

Article history:
Received 25 January 2013
Revised 8 April 2013
Accepted 15 May 2013
Available online 19 June 2013

Keywords:
Peak loads
Failure modes
Upper and lower bounds
Menétrey–Willam-type yield criterion
Haigh–Westergaard coordinates

ABSTRACT

Steel-reinforced concrete beams are analyzed by a plasticity-based approach grounded on nonstandard limit analysis theory. Peak loads and failure modes are predicted by determination of upper and lower bound limits computed with two finite element based numerical procedures. A number of real beams prototypes, belonging to a standard benchmark, are studied and the obtained results are critically discussed outlining possible future developments, strengths and weaknesses of the proposed approach.

1. Introduction and paper outline

Constitutive modeling of concrete structures is a subject of great interest in civil engineering field as witnessed by a huge number of studies that can be found in the relevant literature (see e.g. Chen [7], Hofstetter and Mang [12], Jirásek and Bažant [11], Nielsen and Hoang [25], just to quote a few books dealing, among other, with concrete modeling). Approaches based on very different theories have been proposed, but the most effective are, without doubts, those based on coupling of flow theory of plasticity with fracture or damage mechanics (see e.g. Lubliner et al. [19], Lee and Fenves [17], Grassl and Jirásek [9], Červenka and Papanikolaou [6], Vrech and Etse [36], Zhang et al. [40], Cerioni et al. [5], Zhang and Li [39]). The studies above, mainly oriented to plain concrete, are able to interpret, with good accuracy, most of the complex post elastic phenomena of concrete such as localization, fracturing/damaging mechanisms and so on.

Nevertheless, when dealing with *reinforced* concrete structures, the presence of longitudinal, web or stirrups, reinforcements has a stabilizing influence on concrete fracturing, as noted, for instance, in the remarkable review paper by Bažant [3]. The reinforcements hooping effect on concrete, injects a ductility on the structural elements that renders simpler approaches, as those based on plasticity, quite effective for design purposes (see again Chen [7]). It is in this context that has to be framed the present study, oriented to practical reinforced concrete (RC) structures, and focused on the application of limit analysis, with all the limitations of a plasticity-based approach, as a tool to catch some peculiar aspects of the mechanical behavior of steel-reinforced concrete beams. A number of contributions can be listed to this concern, see e.g. [23,14,10,33], just to quote a few of the more recent ones.

The design methodology expounded hereafter *combines two limit analysis numerical methods* based on the kinematic and the static approach of limit analysis theory, respectively. The former has been deeply treated in a very recent formulation given by the present authors [29]. It is, in practice, an extension to a Menétrey–Willam-type (M–W-type) 3D plasticity model [24], with pressure dependence and cap in compression, of a method for limit analysis originally proposed by Ponter and Co-workers in the 1990s ([30,31]). Such method, known as Linear Matching Method (LMM), [8], gives an *upper bound* on the peak load value of a reinforced concrete element allowing also a prediction of the collapse mode. The LMM hereafter employed is described, for sake of brevity, only in an abridged form being extensively explained in the above quoted paper by the authors. The latter method is, essentially, a generalization to the adopted constitutive model of the Elastic Compensation Method (ECM) due to MacKenzie and Boyle [21] and already applied with success by the authors (within a totally different context) to orthotropic composites structural elements ([26–28]). The application of the ECM in the concrete structures realm is, to the authors' knowledge, a novelty and is

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http://dx.doi.org/10.1016/j.engstruct.2013.05.030

described with greater detail in the following. One of the pursued aims is indeed the extension of this simple numerical method for computing a *lower bound* to the peak load value of a reinforced concrete structure.

The adoption in itself of two well established numerical methods for limit analysis, as the ones here described, is obviously not a novelty even if their reformulation and implementation with reference to a M–W-type model for concrete, which have been completed with the present study, are not-trivial. The dilatancy exhibited by concrete implies the adoption of a nonassociate flow rule and consequently of a *nonstandard limit analysis* approach, the latter furnishing, inevitably, only two bounds to the real collapse load value. Such circumstance justifies the adoption of the two methods. It is worth noting that both methods perform sequences of finite element (FE) based elastic analyses, so resulting easily applicable in other (wider) contexts such those of reinforced concrete framed structures of large dimensions or having an intricate geometry. The original contributions of this work, which is meant to be of practical connotation, can indeed be recognized on two main points, precisely: (i) the *combined* (simultaneous) *use of* the *two methods to bracket the real peak load* values *of RC-elements* and (ii) the *validation* of the promoted procedure *by tackling real large-scale RC-beams* prototypes.

On the other hand, the promoted procedure, whose effectiveness is hereafter verified by comparison with experimental laboratory tests, appears reliable but it is affected by all the congenital limitations of a plasticity based approach. The treatment of post-elastic phenomena that might be exhibited by concrete structures such as: localization, fracturing/damaging mechanisms, creep, and interface problems is not allowed. Indeed, the RC-framed structures commonly used in civil engineering applications, even those of large dimensions, possess a great reserve of ductility (often imposed by the technical standards) that makes applicable and effective a plasticity based analysis. The methodology proposed in the following, oriented to such structures, is then to be viewed as a *design tool* able to give a *preliminary* and useful *information* on the peak load, failure modes, critical zones of reinforced concrete beams and, hopefully, of a RC-framed structure. The analysis at ultimate state of existing structures seems also possible and is the object of an ongoing research. Once a specific structural element or critical zone within a large structure is located, a detailed and more precise incremental nonlinear analysis can be carried out, if necessary, for a deeper comprehension of a suspected more complex failure scenario or for a punctual description of the post-elastic mechanical behavior of such critical or weaker part.

After this introductory section, the adopted Menétrey–Willam-type yield surface, obtained by the homonymous failure surface [24] enriched with a cap in compression [5] and used to model the constitutive behavior of concrete, is expounded in the Haigh–Westergaard coordinates. The theoretical background and the hypotheses assumed for limit analysis are given next together with some details on computational schemes and FE modeling. The systematic analysis of several real prototypes belonging to a classical benchmark on reinforced concrete beams tested up to collapse, [35], is used to validate the whole methodology. Concluding remarks, potentialities and weaknesses of the present study close the paper.

2. Constitutive assumptions for steel and concrete

The present analysis is oriented, as said, to steel-reinforced concrete beams. A perfect bonding is assumed between steel-bars and concrete as well as steel reinforcements exhibit, by hypothesis, an indefinitely elastic behavior while concrete obeys to a plasticity criterion grounded on the three dimensional constitutive model due to Menétrey and Willam [24]. Such assumptions are somewhat different from conventional ones where steel is a plastic material and (plain) concrete a mainly brittle one. Limit analysis however does refer to the structure in the whole and, in the present context, it refers to a standard (commonly used) RC-structure whose ductility is assured by the presence of the reinforcements and whose behavior at ultimate state—at incipient collapse—is dominated by crushing of a confined concrete being the steel bars far from yielding. All the analyzed RC-beam prototypes of the tackled benchmark, addressed hereafter in Section 4 (see also [35]), satisfy this requisite strengthening the assumed hypotheses. Pronounced yielding of the tension reinforcement was not detected in any of the tested beams whose load-deformation response exhibited a fair measure of postpeak ductility. The encouraged limit analysis procedure might certainly be performed within a *FE-layered formulation*, on concrete, governed by the M–W-type criterion, and on steel bars, handled by a Von Mises-type criterion, so ensuring against possible steel bars' yielding at incipient collapse but this point deserves further investigations and will not be addressed here.

The M–W criterion provides a three parameter failure surface having the following expression:

$$f(\xi,\rho,\theta)=\left[\sqrt{1.5}\frac{\rho}{f_c'}\right]^2+m\left[\frac{\rho}{\sqrt{6}f_c'}\,r(\theta,e)+\frac{\xi}{\sqrt{3}f_c'}\right]-c=0, \tag{1}$$

where

$$r(\theta,e)=\frac{4(1-e^2)\cos^2\theta+(2e-1)^2}{2(1-e^2)\cos\theta+(2e-1)[4(1-e^2)\cos^2\theta+5e^2-4e]^{1/2}}, \tag{2}$$

$$m:=3\frac{f_c'^2-f_t'^2}{f_c'\,f_t'}\frac{e}{e+1}. \tag{3}$$

Eq. (1) is expressed in terms of three stress invariants ξ, ρ and θ, known as the Haigh–Westergaard (H–W) coordinates; c is the cohesive strength assumed equal to 1 hereafter; m is the friction parameter of the material depending on the uniaxial compressive strength f_c', on the uniaxial tensile strength f_t' as well as on the eccentricity parameter e governing the convexity and smoothness of the elliptic function $r(\theta,e)$. The eccentricity e describes the out-of-roundness of M–W deviatoric trace and it strongly influences the biaxial compressive strengths of the concrete criterion. The H–W coordinates, namely: the hydrostatic stress invariant, ξ, the deviatoric stress invariant, ρ, the deviatoric polar Lode angle, θ, are related to the stress components by the following relations:

$$\xi=\frac{1}{\sqrt{3}}I_1,\quad I_1=\sigma_{ii}, \tag{4}$$

$$\rho=\sqrt{2\,J_2},\quad J_2=\frac{1}{2}s_{ij}s_{ij}, \tag{5}$$

$$\cos 3\theta=\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}},\quad J_3=\frac{1}{3}s_{ij}s_{jk}s_{ki}, \tag{6}$$

with s_{ij} denoting the deviatoric stress components; i.e., $s_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij}$ being δ_{ij} the Kronecker symbol. It is worth noting that, for $0\leqslant\theta\leqslant\frac{\pi}{3}$, the following relations, between principal stresses of σ_{ij} and H–W coordinates, hold:

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\end{Bmatrix}=\frac{1}{\sqrt{3}}\begin{Bmatrix}\xi\\\xi\\\xi\end{Bmatrix}+\sqrt{\frac{2}{3}}\rho\begin{Bmatrix}\cos\theta\\\cos\left(\theta-\frac{2\pi}{3}\right)\\\cos\left(\theta+\frac{2\pi}{3}\right)\end{Bmatrix}. \tag{7}$$

The sector $0\leqslant\theta\leqslant\frac{\pi}{3}$ confining the values of the Lode angle θ given by $(6)_1$ or, equivalently, by $\cos\theta=(2\sigma_1-\sigma_2-\sigma_3)/2\sqrt{3J_2}$, is