Engineering Structures 56 (2013) 691–697

Contents lists available at SciVerse ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

# Probabilistic characterisation of the length effect for parallel to the grain tensile strength of Central European spruce

J. Kohler [a,*], R. Brandner [b,1], A.B. Thiel [c,2], G. Schickhofer [b,3]

[a] Department of Structural Engineering, Norwegian University of Science and Technology, NTNU, NO 7491 Trondheim, Norway
[b] Institute for Timber Engineering and Wood Technology, Graz University of Technology, Competence Centre holz.bau forschungs gmbh, Inffeldgasse 24/I, 8010 Graz, Austria
[c] Competence Centre holz.bau forschungs gmbh, Inffeldgasse 24/I, 8010 Graz, Austria

ARTICLE INFO

*Article history:*
Received 22 July 2012
Revised 28 May 2013
Accepted 30 May 2013
Available online 4 July 2013

*Keywords:*
Stochastic model
Local strength characteristics
Central European spruce
Length effects
Knot area ratio
Censored data analysis

ABSTRACT

The paper presents a probabilistic model characterising tensile strength parallel to grain of Central European spruce boards without longitudinal joints. The effect of length of members on strength is considered explicitly by considering a member by a serial arrangement of weak sections triggered by major knots of knot clusters. In order to demonstrate the applicability of the proposed model the model parameters are calibrated based on the analysis of three samples of spruce of Switzerland (Mischler-Schrepfer, 2000 [25]) and Austria (Schickhofer and Augustin, 2001 [28]), in total 460 boards tested in full size. This model might serve as basis for studies concerning the length effect of boards relevant for solid timber structures as well as basis for judging the length effect of finger jointed construction timber and glulam, and as starting point for modelling the process of proof loading.



## 1. Introduction

Timber is by nature a very inhomogeneous material. The timber material properties depend on the specific wood species, provenience and on the local growing conditions of every single part of a tree. Timber is a rhombic anisotropic, roughly orthotropic material, i.e. it consists of "high strength" fibres which are predominantly orientated along the longitudinal axis of trees embedded in a "low strength" matrix. After a log is sawn into pieces of structural timber, irregularities like grain direction, knots and fissures, become, in addition to the roughly orthotropic characteristics mentioned above, highly decisive for the load bearing behaviour of a timber structural element. Strength and stiffness of timber, consequently, can only be considered relative to a particular loading mode, the geometry of the corresponding timber element and the distribution of irregularities in the wood fibre matrix, i.e. strength and stiffness of timber is always considered on an element level and not on a material level. Furthermore, the load bearing behaviour of timber structural elements is highly sensitive to load duration and climate constraints.

The particular character of structural timber constitutes a major challenge for researchers and engineers with the aim to use timber as a safe and efficient load bearing material in construction. One major topic that is continuously discussed within the research community is the appropriate representation of size effects on strength of solid timber. For most loading modes as tension parallel or perpendicular to grain, shear or bending, timber predominately presents brittle failure behaviour. A (perfect) brittle material is defined as a material that fails if a single particle fails; see e.g. [3]. The strength of the material is thus governed by the strength of the "weakest" particle; therefore the model for ideal brittle materials is also called the "weakest link model", [33].

Over the last 40 years, the perfect brittle material model has been widely applied to study the tension, bending, bending shear and compression strength variability of various wood products including sawn and glued laminated timber. Some studies have shown the conformity between the ideal brittle material model assumptions and experimentally observed length and depth effects in tension and/or bending (e.g. [4,23]) or for the prediction of the material behaviour in tension perpendicular to grain (e.g. [1,8,9]. There is an ongoing list of further literature, and many applications of the perfect brittle material model for timber can be found. However, the results of these investigations are delivering partly contrary results [5,20]; contrary in regard to the value of quantified parameters, as e.g. the scale parameter of the Weibull model, but

* Corresponding author. Tel.: +47 (0) 735 94517.
*E-mail addresses:* jochen.kohler@ntnu.no (J. Kohler), reinhard.brandner@tugraz.at (R. Brandner), alexandra.thiel@tugraz.at (A.B. Thiel), gerhard.schickhofer@tugraz.at (G. Schickhofer).
*URLs:* http://www.ntnu.no (J. Kohler), http://www.lignum.at, http://www.holzbauforschung.at (R. Brandner), http://www.holzbauforschung.at (A.B. Thiel), http://www.lignum.at, http://www.holzbauforschung.at (G. Schickhofer).
[1] Tel.: +43 (0) 316 873 4605; fax: +43 (0) 316 873 4619.
[2] Tel.: +43 (0) 316 873 4606; fax: +43 (0) 316 873 4619.
[3] Tel.: +43 (0) 316 873 4600; fax: +43 (0) 316 873 4619.

0141-0296/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.engstruct.2013.05.048

also in regard to the observed phenomena (e.g. [4,7,12,34]. This can be explained by the variability of the timber material at a macro scale, i.e. different species and grades exhibit different size effects [27]; but also by the fact that the assumptions underlying the theory of ideal brittle fracture are not strictly fulfilled when considering timber. Timber is referred to as a rhombic anisotropic material; strength and stiffness properties are depending on the stress direction of local coordinate system in a timber solid. Therefore the theory can only be used for individual loading modes for which the stress direction can assumed to be constant, i.e. timber components loaded to tension perpendicular or parallel to the grain, shear along the grain or pure bending of beam shaped timber specimen. The latter loading mode includes tension and compression and has to be considered with caution because the failure mode in compression is not brittle. The material irregularities are in the scale of $\mu m$ if the fibre configuration is considered or in the scale of *dm* when considering major defects as knots and grain deviations. The latter defects are almost in the same scale as structural components itself, which is in conflict to the basic assumption of the brittle material model, i.e. a material with a large number of defects, identical distributed and independent. To overcome the violation of model boundaries the weakest link theory of [33] was modified to account for the anisotropy of timber (e.g. [2,23] and by additional terms found by multiple regression analysis to account for the influence of growth characteristics (e.g. [7,12]. Later first considerations on spatial correlation of local strength properties were presented in [34] and by means of bi-modal distributions.

An alternative approach for the representation of size effects for solid timber is first presented by Riberholt and Madsen [26] and the followed up by e.g. [5,13,14,17,19–22,29–32,35]. The basic idea is that a timber structural element fails predominantly at locations with major irregularities due to knot clusters, or "weak zones" and that knot clusters or "weak zones" are more or less regularly distributed along the longitudinal axis of a structural element. These second- and third-order hierarchical models allow explicitly accounting for the spatial correlation of local strength and other physical properties as well as for the spatial distribution and magnitude of flaws indicating these properties locally. These hierarchical models are in particular of importance for hierarchically structured natural materials like timber. A more detailed and comprehensive review can be found e.g. in [5].

Following that idea several studies concentrated on the statistical representation of the strength and the occurrence of weak zones along a structural element. For example Riberholt and Madsen [26] observed a remarkable variation of knot zone distance (KZD) even within the same timber species. For spruce (*Picea abies* (L.) *Karst.*) a mean KZD in the range of 300–500 mm was found together with a positive dependency on strength class. Czmoch et al. [11] state in reference to Colling and Dinort [10] as well as Jönsson and Östlund [18] that the KZD is independent of the strength class and can be expected to be 500 mm in spruce. Williamson [34] concluded that the size of flaws (e.g. knots) depends on strength class but the quantity not. Källsner et al. [19] observed 6–7 weak zones (knot clusters) in boards of 3.5 m length. Isaksson [17] report a mean KZD of 450–500 mm and observed that knot area ratio (KAR) values appear to be independent of growth region, board width and strength class. Especially softwood species like spruce (*P. abies* (L.) *Karst.*) are characterised by regularly spaced knot clusters, whereby the distance between these clusters corresponds to the yearly primary longitudinal growth increment.

In the present paper the idea of representing a timber structural element as a sequence of weak sections is followed further and a probabilistic model for the timber tension strength is developed. The model is representing the length effect on strength explicitly. It is demonstrated how the parameters of the model can be calibrated on basis of standard tension test data.

## 2. Probabilistic model of the length effect for parallel to the grain tensile strength

A tension structural element is represented as a serial system of sections, whereas it is distinguished between "strong" and "weak" sections. Weak sections coincide with the presence of knots and knot clusters. It is assumed that the tension capacity of a tension structural element is governed entirely by the weak sections. This includes the assumption that the structural element contains at least one weak section.

The strength of a single weak section might be represented by a random variable *R*, with

$$R = X + Y \tag{1}$$

whereas *X* is a random variable representing the variation of the mean weak section strength of a component consisting of several weak sections. *Y* is a zero mean random variable representing the random deviation of *R* from *X*. Thus, Eq. (1) is representing a hierarchical model for the tension strength of one weak section, whereas it is explicitly differentiated between "within" (*Y*) and "between" (*X*) member variation. A hierarchical model was previously used for timber bending strength in [17,26] and [13,14]. A hierarchical model formulation was extended to also represent "between" sample variation in [20].

Based on Eq. (1) the tension strength of a timber structural element containing $k \geqslant 1$ weak sections can be written as

$$R_{element} = R_{\min,k} = X + \min_{k}(Y_1, Y_2, \ldots, Y_k) \tag{2}$$

i.e. the tension strength of the element is equivalent to the tension strength of the weakest section.

In summary, the outlined representation includes the following main assumptions:

- Failure takes place at weak sections exclusively.
- Failure is originated at one weak section.
- Every structural element contains at least one weak section.
- The correlation between different weak sections of one element is independent from the distance between the different weak sections (→equicorrelation).

As in [13,14] for the timber bending strength, the probability distribution function of the tension strength of one board with $K = k \geqslant 1$ weak sections, $R_{element,k}$, may be derived as

$$F_{R_{element,k}}(r|K=k) = \int_{-\infty}^{\infty} F_Y^{min,k}(r|x) f_X(x)dx \Longleftrightarrow F_{R_{element,k}}(r|K=k) = \int_{-\infty}^{\infty} (1-(1-F_Y(r|x)))^k f_X(x)dx \tag{3}$$

The tension strength of timber parallel to the grain direction, conventionally, is represented with a lognormal distributed random variable. If Eq. (1) is rewritten as

$$\ln(R) = Y + X \tag{4}$$

*R*, the representation of the tension strength of a weak section is lognormal distributed if *Y* and *X* are normal distributed random variables. Then, Eq. (3) can be rewritten accordingly, as

$$F_{\ln(R_{element,k})}(\ln(r)|K=k) = \frac{1}{\sigma_X}\int_{-\infty}^{\infty}\left(1-\Phi\left(-\frac{\ln(r)-x}{\sigma_Y}\right)^k\right)\varphi\left(\frac{x-\mu_X}{\sigma_X}\right)dx \tag{5}$$

where $\varphi(.)$ is the standardised normal density function, $\Phi(.)$ is the standardised normal distribution function, $\mu_X$ is the mean value of *X* and $\sigma_X$ and $\sigma_Y$ are the standard deviations of *X* and *Y* (note that $\mu_Y$ is equal to 0 according to the definition of the hierarchical model).