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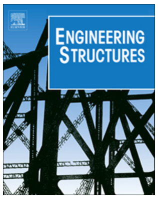

Nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments

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ABSTRACT

This paper studies the behaviors of large amplitude vibration, nonlinear bending and thermal postbuckling of nanocomposite beams reinforced by single-walled carbon nanotubes (SWCNTs) resting on an elastic foundation in thermal environments. Two types of carbon nanotube-reinforced composite (CNTRC) beams, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The material properties of FG-CNTRCs are assumed to be graded in the beam thickness direction, and are estimated through a micromechanical model. The motion equations of a CNTRC beam on an elastic foundation are derived based on a higher order shear deformation beam theory. The thermal effects are also included in the motion equations and the material properties of CNTRCs are assumed to be temperature-dependent. Numerical studies are carried out for the nonlinear vibration, nonlinear bending and thermal postbuckling of CNTRC beams resting on Pasternak elastic foundations under different thermal environmental conditions. It is found that a CNTRC beam with intermediate CNT volume fraction does not necessarily have intermediate nonlinear frequencies, buckling temperatures and thermal postbuckling strengths. Thermal postbuckling path of unsymmetric FG-CNTRC beams is no longer the bifurcation type.

1. Introduction

Carbon nanotubes (CNT) possess exceptional mechanical, thermal and electrical properties and are used as significant reinforcement materials for high performance structural composites with substantial application potentials [1–3]. Carbon nanotube-reinforced composites (CNTRCs) have advanced mechanical properties such as high strength, high stiffness and light weight which can be applied as layers in advanced laminated structures. Recent researches on CNTRCs [4–7] observed that only a low percentage of CNTs (2–5% by weight) can be added to the composites as more volume fraction in CNTRCs can actually cause the deterioration of their mechanical properties [8]. Shen [9] proposed to apply the functionally graded (FG) concept to CNTRCs in order to effectively make use of the low percentage of CNTs in the composite. He studied the nonlinear bending behavior of CNTRC plates with a linear distribution of CNTs along the thickness direction of the plates and observed that the load-bending moment curves of the plates can be considerably improved through the use of a functionally graded distribution of aligned CNTs in the matrix. Shen and his co-authors [10–14] further extended the study to the postbuckling and nonlinear vibration of CNTRC plates and shells and highlighted the influence of the FG-CNT distribution patterns on the mechanical behaviors of the CNTRC structures. The concept of functionally graded nanocomposites is strongly supported by a recent publication [15] in which a functionally graded CNT reinforced aluminum matrix composite was fabricated by a powder metallurgy route. Consequently, investigations on bending, buckling and vibration of CNTRC structures are recently emerged as an interesting field of study [16–20].

Several studies have been reported on the bending, buckling and vibration of CNTRC beams based on Euler–Bernoulli beam theory, Timoshenko beam theory and higher order shear deformation beam theory [21–26]. Among those, Ke et al. [21] investigated the nonlinear free vibration of functionally graded CNTRC Timoshenko beams. They found that both linear and nonlinear frequencies of functionally graded CNTRC beams with symmetrical distribution of CNTs are higher than those of beams with uniform or unsymmetrical distribution of CNTs. This work was then extended to the cases of functionally graded CNTRC Euler–Bernoulli beams with piezoelectric layers by Rafiee et al. [22] and dynamic stability of functionally graded CNTRC Timoshenko beams by Ke et al. [23]. Yas and Heshmati [24] presented free vibration and linear buckling of functionally graded CNTRC Timoshenko beams on an elastic foundation by using the differential quadrature method. They [25] also presented a dynamic analysis of functionally graded CNTRC beams under the

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action of moving load by using the finite element method (FEM). Recently, Wattanasakulpong and Ungbhakorn [26] presented linear bending, buckling and vibration of CNTRC beams resting on an elastic foundation based on a higher order shear deformation beam theory. Like in the case of functionally graded ceramic–metal beams with simply supported boundary conditions, the bifurcation buckling of simply supported functionally graded CNTRC beams does not exist due to the stretching–bending coupling effect. Therefore, the bifurcation solutions for simply supported unsymmetric functionally graded CNTRC beams subjected to in-plane compression and temperature variation may be physically incorrect [24], unless the compressive load or the stress resultant caused by temperature rise is applied on the physical neutral surface of the CNTRC beam.

In the present work, we focus our attention on the nonlinear free vibration, nonlinear bending and thermal postbuckling of CNTRC beams resting on an elastic foundation in thermal environmental conditions. Two types of CNTRC beams, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The motion equations are based on a higher order shear deformation beam theory and von Kármán-type nonlinear strain–displacement relationships. The beam-foundation interaction and thermal effects are also included. The material properties of CNTRCs are assumed to be temperature-dependent. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. Two ends of the beam are assumed to be simply supported and in-plane boundary conditions are assumed to be immovable. The nonlinear vibration characteristics, nonlinear bending and thermal postbuckling behaviors of CNTRC beams resting on an elastic foundation under different sets of thermal environmental conditions are presented and discussed in details.

2. Effective material properties of functionally graded CNTRCs

We assume that the CNTRC layer is made from a mixture of aligned single-walled carbon nanotubes (SWCNTs) and matrix which is assumed to be isotropic. The SWCNT reinforcement is either uniformly distributed or functionally graded along the thickness direction of a CNTRC structure. At the nanoscale, the structure of the carbon nanotube strongly influences the overall properties of the composite. Several micromechanical models have been developed to predict the effective material properties of CNTRCs, for instance, the Mori–Tanaka model [27,28] and the Voigt model as the rule of the mixture [29,30]. The Mori–Tanaka model is applicable to micro-particles and the rule of mixture is simple and convenient to predict the global material properties and responses of the CNTRC structures. At nanoscale both Mori–Tanaka and Voigt models need to be extended in order to include the small scale effect. It has been shown that the Voigt and Mori–Tanaka models have the same accuracy in predicting the buckling and vibration characteristics of functionally graded ceramic–metal beams [31], plates [32] and shells [33]. According to the extended rule of mixture, the effective Young's modulus and shear modulus of CNTRCs can be expressed as [9]

$$E_{11} = \eta_1 V_{CN} E_{11}^{CN} + V_m E^m \tag{1a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CN}}{E_{22}^{CN}} + \frac{V_m}{E^m} \tag{1b}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CN}}{G_{12}^{CN}} + \frac{V_m}{G^m} \tag{1c}$$

where E_{11}^{CN}, E_{22}^{CN} and G_{12}^{CN} are the Young's and shear moduli of the CNTs, E^m and G^m are the corresponding properties for the matrix, and the η_j $(j = 1,2,3)$ are the CNT efficiency parameters, respectively. In addition, V_{CN} and V_m are the volume fractions of the CNT and the matrix, which satisfy the relationship of $V_{CN} + V_m = 1$.

It has been reported that the load transfer between the nanotube and polymeric phases is less than perfect (e.g. the surface effects, strain gradients effects, intermolecular coupled stress effects, etc.). Hence, we introduce the CNT efficiency parameter η_j $(j = 1,2,3)$ into Eq. (1) to consider the small scale effect and other effects on the material properties of CNTRCs. The values of η_j will be determined later by matching the elastic moduli of CNTRCs predicted by the MD simulations with the prediction of the extended rule of mixture in Eq. (1).

Unlike the functionally graded ceramic–metal materials [34], to avoid abrupt change of the material properties in the functionally graded CNTRCs, a linear variation of the CNT volume fraction in the thickness beam direction is assumed and can readily be achieved in practice [15]. Consequently, we assume the volume fraction V_{CN} for the top face sheet as

$$V_{CN} = 2\left(\frac{t_1 - Z}{t_1 - t_0}\right) V_{CN}^* \tag{2a}$$

and for the bottom face sheet follows as

$$V_{CN} = 2\left(\frac{Z - t_2}{t_3 - t_2}\right) V_{CN}^* \tag{2b}$$

in which

$$V_{CN}^* = \frac{w_{CN}}{w_{CN} + (\rho^{CN}/\rho^m) - (\rho^{CN}/\rho^m) w_{CN}} \tag{2c}$$

where w_{CN} is the mass fraction of CNTs, and ρ^{CN} and ρ^m are the densities of CNT and matrix, respectively. It is evident that $V_{CN} = 2V_{CN}^*$, when $Z = t_0$ (top surface) and $Z = t_3$ (bottom surface) and $V_{CN} = 0$ when $Z = t_1$ and $Z = t_2$. In such a way, the two cases of uniformly distributed (UD), i.e. $V_{CN} = V_{CN}^*$, and functionally graded (FG) CNTRCs will have the same value of mass fraction of nanotubes. It is noted that unlike the elastic properties, the load transfer between the CNT and matrix does not affect the density of CNTRCs at a given location. Therefore, no CNT efficient parameter is needed to modify the CNTRC mass density which is defined by $\rho = V_{CN}\rho^{CN} + V_m\rho^m$. As the same in [14], three types of FG-CNTRCs, i.e. FG-V, FG-Λ and FG-X, may be considered. For FG-V, Eq. (2a) is available, and for FG-Λ, Eq. (2b) is available, and for FG-X, both Eqs. (2a) and (2b) are adopted.

Similarly, the effective thermal expansion coefficients in the longitudinal and transverse directions can be expressed by the Shapery model [35]

$$\alpha_{11} = \frac{V_{CN} E_{11}^{CN} \alpha_{11}^{CN} + V_m E^m \alpha^m}{V_{CN} E_{11}^{CN} + V_m E^m} \tag{3a}$$

$$\alpha_{22} = (1 + \nu_{12}^{CN}) V_{CN} \alpha_{22}^{CN} + (1 + \nu^m) V_m \alpha^m - \nu_{12}\alpha_{11} \tag{3b}$$

where α_{11}^{CN}, α_{22}^{CN} and α^m are thermal expansion coefficients, and ν_{12}^{CN} and ν^m are Poisson's ratios, respectively, of the carbon nanotube and matrix. Note that α_{11} and α_{22} are also graded in the Z direction. It is assumed that the material properties of nanotube and matrix are functions of temperature, so that the effective material properties of FG-CNTRCs, like Young's modulus, shear modulus and thermal expansion coefficients, are functions of temperature T and position Z. The Poisson's ratio depends weakly on temperature change and position and is expressed as

$$\nu_{12} = V_{CN}^* \nu_{12}^{CN} + V_m \nu^m \tag{4}$$

3. Governing equations

Consider a uniform beam of length L, width b, and thickness h, with two pinned ends and resting on a two-parameter elastic