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An analytical solution for an inflated orthotropic membrane tube with an arbitrarily oriented orthotropy basis

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ABSTRACT

In this paper, an analytical solution is proposed for a pressurized cylindrical tube made of an orthotropic membrane, with the orthotropy directions oriented at an arbitrary angle, not necessarily parallel to the tube axis. The formulation is made in the framework of finite deformations and results in a system of three non-linear equations, giving the final radius, length and rotation of the cross-sections of the tube, in the deformed configuration. The set of equations is solved for balanced and unbalanced materials, with various internal pressures and orientations of the membrane. The numerical results obtained are shown to agree very well with those of an independent finite element code.

1. Introduction

Tensile fabrics have been widely used in textile architecture for more than half a century. Nowadays, very large structures are built as assemblies of pieces of fabrics and among the materials used, coated fabrics occupy a major place due to their interesting mechanical properties: they are light, they can be easily folded and deployed, and they are not too expensive to manufacture.

These fabrics are typically made of woven yarns encased in a PVC coating and therefore present an anisotropic behavior due to the warp and weft threads. Since they have very low bending and compressive stiffness, they can only be used in tension. In order to use them under other loading states, it is necessary to induce a pre-stress by means of air pressure, as in the so-called air-supported and air-inflated structures. The subsequent discussion will be limited to air-inflated structures only, also known as inflatable or pneumatic structures, which are the scope of the present paper.

The inflatable tube is one of the most simple elements in inflatable structures technology. Such a tube can be used either as a single beam or assembled with others in inflatable frames and more complex structures.

The study of inflatable beams necessitates to distinguish two successive stages: (i) in the first one, the beam is subjected to a sufficiently high internal pressure, so as to induce the pre-stress in the membrane and provide the beam with a bearing capacity, (ii) in the second stage, the beam can be subjected to other external loadings which can be a combination of tensile, compressive, bending or twisting loads.

Almost all the existing studies on inflatable tubes dealt with bending or torsion loadings. Pioneering theoretical works are due to Comer and Levy [1], Douglas [2] and Webber [3], who investigated isotropic beams using Euler–Bernoulli's kinematics. Main et al. [4,5] undertook experimental works on the bending of inflatable beams and took account of the biaxial state in their analysis. The shear effect in inflatable beams was considered by Steeves [6] who developed a theory based on the minimum potential energy principle and gave the solution in terms of Green functions, and by Fichter [7] who used Timoshenko's kinematics and obtained load–deflection formulae where the pressure appears explicitly in the shear stiffness term. Adopting the same kinematics, Wielgosz and Thomas [8,9] derived analytical solutions for inflated panels and tubes, by writing the equilibrium equations in the pre-stressed state, and taking into account the pressure as a following force. Le van and Wielgosz [10] improved Fichter's theory by using the total Lagrangian form of the virtual work principle with finite displacements and rotations. After linearizing the resulting nonlinear equations, they obtained linear formulae for isotropic beams where the pressure appears explicitly both in the bending and shear stiffnesses. Recently, Apedo et al. [11] went further using a 3D Timoshenko's kinematics and derived their solutions for an orthotropic fabric in finite displacements and small rotations. This work was next extended to the buckling of an orthotropic beam by Nguyen et al. [12]. Refinements of the previous formulations enabled Nguyen et al. [13] to derive the governing nonlinear equations for inflatable orthotropic beams and simple formulae for the deflection and rotation.

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In all the above-mentioned papers, the geometry of the inflated beam is either assumed to be known or computed by means of linear elasticity formulae, as in Le van and Wielgosz's paper [10] for isotropic materials, and in Apedo et al's paper [11] for orthotropic materials. Whereas such linear formulae are sufficient in the context of small deformations, they should be replaced by more accurate formulae when there are finite deformations, as is the case nowadays with modern fabrics which are able to bear very high tensions.

The present work is devoted to the above-mentioned first stage where the beam is subjected to an internal pressure only, with the purpose of obtaining an analytical solution for a pressurized membrane tube made of an orthotropic material. The main features of the paper are: (i) the warp or weft direction of the membrane can be oriented at an arbitrary angle, not parallel to the tube axis (for convenience, use is made of the terms 'warp' and 'weft' to designate the in-plane orthotropy directions of the membrane); and (ii) the obtained equations hold in finite deformations, in particular finite rotations of the tube.

The paper is organized as follows. In Section 2, the problem will be formulated in finite deformations for a finite-thickness orthotropic tube and the equations will be derived considering a very small thickness with respect to the tube radius so that the tube becomes a membrane tube. This will result in a small system of three nonlinear equations, which can easily be solved by an iterative Newton-type scheme to get the final geometry as well as the stresses in the pressurized tube. In Section 3, the influence of the internal pressure and the orthotropy directions will be studied for two materials, one is balanced and the other unbalanced. It will be shown that when the warp or weft direction of the membrane is not parallel to the tube axis, the tube undergoes a rotation around its axis, rotation which varies as a nonlinear function of the internal pressure. Eventually, in Section 4, comparison with finite element results will be made in order to validate the proposed theory.

Note that we consider an orthotropic *homogeneous* membrane, rather than a fabric which is inhomogeneous due to its complex microstructure. Phenomena specific to fabrics such as kinematics of crimp interchange, shear jamming and yarn–yarn friction, will not be considered as they are out of the scope of the paper. However, one may expect that the results obtained can be used for fabrics to a certain extent.

2. Analytical solution for an inflated orthotropic membrane tube

2.1. Definition of the problem

The problem is formulated in finite deformations and one has to distinguish between the reference and the final configurations of the tube. Let the current position $\mathbf{x}$ of a particle of the tube in the actual state be defined by the cylindrical coordinates r, θ, z, so that $\mathbf{x} = r\mathbf{e}_r(\theta) + z\mathbf{e}_z$. The current cylindrical basis at point $\mathbf{x}$ is denoted $\mathbf{b} \equiv (\mathbf{e}_r(\theta), \mathbf{e}_\theta(\theta), \mathbf{e}_z)$, with $\mathbf{e}_r$ and $\mathbf{e}_\theta$ parallel to the radial and circumferential directions, respectively.

The position $\mathbf{X}$ of the same particle in the reference state is defined by the initial values of r, θ and z, denoted by R, Θ and Z, so that $\mathbf{X} = R\mathbf{e}_r(\Theta) + Z\,\mathbf{e}_z$. The reference cylindrical basis at point $\mathbf{X}$ is $\mathbf{B} \equiv (\mathbf{e}_r(\Theta), \mathbf{e}_\theta(\Theta), \mathbf{e}_z)$.

The reference geometry is a thick-walled tube of axis $O\mathbf{e}_z$, inner radius A, outer radius B, thickness $H = B - A$, length L, and closed at the ends $Z = 0$ and $Z = L$, as shown in Fig. 1.

It is assumed that the tube is made of an orthotropic material with the orthotropy basis $(\mathbf{e}_n, \mathbf{e}_\ell, \mathbf{e}_t)$ in the reference configuration. The normal direction $\mathbf{e}_n$ is equal to $\mathbf{e}_r(\Theta)$ and the angle (around $\mathbf{e}_r(\Theta)$) between the longitudinal direction $\mathbf{e}_\ell$ and the tube axis $\mathbf{e}_z$ is denoted α, $0 \leqslant \alpha \leqslant 180°$.

The tube is free of stress in the reference configuration and let it be subjected to an internal pressure p.

2.2. Deformation

It is assumed that the cross-sections of the pressurized tube remain planar and perpendicular to axis $O\mathbf{e}_z$, in such a way that the tube remains cylindrical in the deformed state. The deformed geometry is defined by the inner radius a, outer radius b, thickness $h = b - a$ and length ℓ, which correspond to their initial values A, B, H and L, respectively. The deformation of the tube is then defined by the following relation giving the final cylindrical coordinates r, θ, z as functions of the initial coordinates R, Θ, Z:

$$r = r(R) \quad \theta = \Theta + \beta(R,Z) \quad z = z(Z) \tag{1}$$

Functions $r(R)$ and $z(Z)$ correspond to the radial and axial displacements, whereas function $\beta(R,Z)$ has been introduced in order to represent the circumferential displacement or the rotation of the cross-sections around the tube axis, due to the fact that the orthotropy directions do not coincide with the cylindrical axes. The functions $\beta(R,Z)$ and $z(Z)$ have to satisfy $\beta(R,0) = 0$ and $z(0) = 0$ if one considers that the end section $Z = 0$ does not rotate.

2.3. Deformation gradient – Strains

Since the current position $\mathbf{x}$ of a particle of the tube is resolved in terms of the reference basis $\mathbf{B}$ via $\mathbf{x} = r\cos\beta\ \mathbf{e}_r(\Theta) + r\sin\beta\ \mathbf{e}_\theta(\Theta) + z\ \mathbf{e}_z$, the matrix of the deformation gradient tensor can be expressed in basis $\mathbf{B}$ as

$$Mat(\mathbf{F};\mathbf{B}) = \begin{bmatrix} k_r\cos\beta - r\sin\beta\beta_{,R} & -k_\theta\sin\beta & -r\sin\beta\beta_{,Z} \\ k_r\sin\beta + r\cos\beta\beta_{,R} & k_\theta\cos\beta & r\cos\beta\beta_{,Z} \\ 0 & 0 & z_{,Z} \end{bmatrix} \quad \text{with}$$

$$k_r \equiv \frac{dr}{dR}, \quad k_\theta \equiv \frac{r}{R} > 0 \tag{2}$$

The bijectivity condition $J = det\mathbf{F} = k_r k_\theta z$, $Z > 0$ leads to

$$k_r z_{,Z} > 0 \tag{3}$$

The matrix of the Green strain tensor $\mathbf{E} = (\mathbf{F}^T\mathbf{F} - \mathbf{I})/2$ in the reference basis is derived from Relation (2):

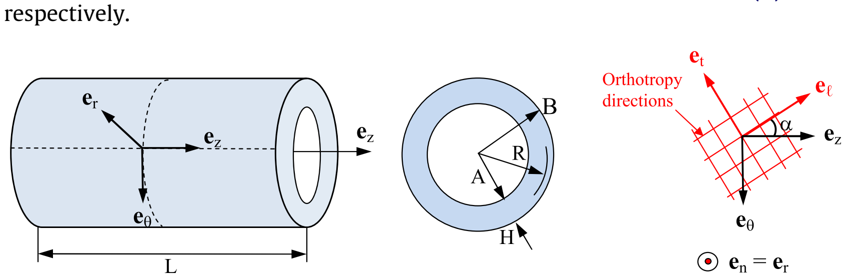

Fig. 1. Reference geometry of the tube.