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Strain-based calculation model for centrically and eccentrically loaded timber columns

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ABSTRACT

The behaviour of timber members subjected to axial compression or combined axial compression and bending is characterised by the non-linear increase of the deformation due to the increasing eccentricity of the axial load and due to the non-linear material behaviour. The paper presents a strain-based model taking into account these effects.

Design approaches given in timber structures design codes are compared and differences in the results obtained with the different approaches are identified. Furthermore, a strain-based model to analyse the load-bearing capacity of centrically and eccentrically loaded timber columns is described and its power is assessed. It is shown that in particular the non-linear behaviour of timber when subjected to compression parallel to the grain considerably influences the load-bearing capacity.

The model is validated on the basis of experimental investigations on solid Norway spruce beams loaded in combined axial compression and bending. A good agreement was found between the estimated values using the strain-based model and the experimentally derived values.

A comparison of the model with the design approaches given in the codes shows that the load-bearing capacity can be overestimated under certain conditions. Finally, it is illustrated how the design approaches can be modified in order to reach a more consistent design.

1. Introduction

Axial compression or combined axial compression and bending are encountered in many types of timber members such as columns, frame structures or compression members of truss girders. The behaviour of these structural members is primarily characterised by the non-linear increase of the deformation due to the increasing eccentricity of the axial load (P-delta effect). In addition to this geometric non-linear behaviour, the non-linear material behaviour of timber members subjected to compression parallel to the grain has to be accounted for.

The influence of the P-delta effect on the load-bearing capacity of timber members subjected to axial compression was investigated first by Tetmajer [1] in 1896. Tetmajer's studies set up the basis for the design of timber members subjected to axial compression for a long time. Tests performed by Larsen and Pedersen [2] confirmed the results obtained by Tetmajer. The experimental investigation showed the great influence of the varying material properties on the load-bearing capacity. In order to account for these variations and hence to estimate the resistance of glued laminated timber members subjected to compression more accurately, Blaß [3,4] performed Monte Carlo simulations. The buckling curves given in different design codes [5–7] were derived from these investigations. For timber members subjected to combined axial compression and bending, Buchanan [8,9] developed a numerical model capable of investigating the influence of the non-linear material behaviour on the moment – axial force interaction. In addition Buchanan investigated the influence of the size of the member.

Current design codes (e.g. Eurocode 5 [5] or the Swiss national code for the design of timber structures SIA 265 [6]) provide two different approaches for the design of centrically and eccentrically loaded timber columns:

– a simplified calculation model based on the Effective Length Method (ELM),
– 2nd order analysis of the structure.

In ELM, the buckling problem of a structural system is reduced to that of an equivalent simply supported (pinned) column. The 2nd order analysis of the structure is a method which takes into account the non-linearity by studying the equi-

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librium of the deformed structural system. In general, non-linearity caused by the increasing eccentricity of the external load as well as non-linearity caused by the non-linear material behaviour of timber subjected to compression should be considered. However, the 2nd order analysis is often understood as a theory based on linear elastic material behaviour and the effects caused by the non-linearity of the material are neglected. Even the design codes [5–7] only provide rules for this 2nd order linear elastic analysis of the structure. In this paper, a clear distinction between the 2nd order linear elastic analysis and the generalised 2nd order analysis is made.

The two approaches (ELM and 2nd order linear elastic analysis of the structure) given in the codes are not consistent and can lead to different results. This situation led to controversial discussions in the scientific community [10–13]. The discussion in particular showed that there are inconsistencies concerning the consideration of the effect of moisture content (MC) and duration of load (DOL) as well as inconsistencies concerning the implementation of the 2nd order linear elastic analysis in the design codes. While recent research on the load-bearing behaviour of timber members subjected to axial compression or combined axial compression and bending was mainly focused on MC and DOL [10–12,14,15], this publication deals with the influence of the non-linear material behaviour and with the implementation of 2nd order linear elastic analysis in the design codes such that there are only minor differences between 2nd order linear elastic analysis and the Effective Length Method. Hence, the results presented here are only valid for short-term response under load at constant interior climate. In fact, MC and DOL and in particular the creep behaviour and the climate take a major impact on the load-bearing behaviour of timber columns and should also be considered for the design of timber members subjected to compression or combined compression and bending [16–18].

2. Design of timber members in compression parallel to the grain

The current design codes such as Eurocode 5 EN 1995-1-1 [5], the Swiss design code SIA 265 [6] or the withdrawn German code DIN 1052 [7] provide two different approaches for the design of timber member subjected to either compression parallel to the grain or combined axial compression and bending. In general, the Effective Length Method is used for simple design situations while the 2nd order linear elastic analysis of the structure provides some advantages for more complex design situations.

2.1. Effective Length Method (ELM)

The simplified calculation model is based on the Effective Length Method (ELM). The buckling problem of a structural system is reduced to that of an equivalent simply supported (pinned) column [19].

For the design, the internal forces and moments are calculated based on a simple 1st order analysis and the non-linear P-delta effect is taken into account by means of a buckling factor k_c. This factor describes the ratio between the axial stress at buckling failure of a member subjected to axial compression and its compressive strength parallel to the grain. k_c depends on the effective length of the structural system which can be expressed by the slenderness ratio λ (Eq. (1)).

$$\lambda = \frac{\ell_{cr}}{i} \tag{1}$$

$$i = \sqrt{\frac{I}{A}} \tag{2}$$

where λ is the slenderness ratio; ℓ_{cr} is the effective length; i is the radius of gyration; I is the 2nd moment of inertia and A is the area of the cross-section.

The buckling factor k_c as used in [5–7] is based on extensive investigations performed by Blaß [3]. In order to determine the characteristic (i.e. 5th percentile) load-bearing capacity of timber columns a Monte Carlo simulation technique was used. The numerical model and the parameter study considered the P-delta effect, the variability of the strength and the stiffness properties within the timber members, the geometric imperfection of the timber members and the non-linear material behaviour of timber when subjected to compression parallel to the grain and bending.

For the ultimate limit state analysis, the design codes [5–7] recommend to use a linear interaction model for combined axial compression and bending as given in Eq. (3). In this interaction model, the buckling factor k_c is used to reduce the compressive strength parallel to the grain of the timber member in order to account for probable buckling.

$$\frac{\sigma_{c,0,d}}{k_c \cdot f_{c,0,d}} + \frac{\sigma_{m,I,d}}{f_{m,d}} \leqslant 1 \tag{3}$$

$\sigma_{c,0,d}$ is the design value of the acting compressive stress parallel to the grain; k_c is the buckling factor; $f_{c,0,d}$ is the design compressive strength parallel to the grain; $\sigma_{m,I,d}$ is the design value of the acting bending stress derived by means of a 1st order structural analysis and $f_{m,d}$ is the design bending strength.

2.2. 2nd Order linear elastic analysis of the structure

As an alternative to the calculation model based on the Effective Length Method, timber members subjected to axial compression or combined axial compression and bending can be designed by performing a 2nd order linear elastic analysis of the structure. The 2nd order linear elastic analysis is a method which takes into account the geometric non-linearity by studying the equilibrium of the deformed structural system. An initial deformation is introduced into the calculation in order to account for the geometric imperfection of the structural member as e.g. deviation from a perfectly straight shape.

For a simply supported, axially loaded column the 2nd order linear elastic analysis can easily be performed, assuming sinusoidal distributed initial deformations. The initial deformation in combination with the axial load leads to an initial bending moment M_I. The P-delta effect causes a magnified moment M_{II}. M_{II} can be calculated by multiplying the initial bending moment M_I by a magnification factor μ [20]:

$$M_{II} = M_I \cdot \mu \tag{4}$$

$$\mu = \frac{1}{1 - \frac{N}{N_{\text{Euler}}}} \tag{5}$$

N_{Euler} is the Euler buckling load:

$$N_{\text{Euler}} = \frac{\pi^2 \cdot EI}{\ell_{cr}^2} \tag{6}$$

M_{II} is the magnified bending moment (2nd order linear elastic theory, deformed structural system); M_I is the initial bending moment (1st order theory, undeformed structural system); μ is the magnification factor; N is the normal force acting on the column; N_{Euler} is the Euler buckling load; E is the modulus of elasticity (MOE); I is the 2nd moment of inertia and ℓ_{cr} is the effective length.

Timber members subjected to combined axial compression and bending tend to develop non-linear deformations of the compression zone before failure occurs. This non-linearity leads to a curved shape of the moment – axial force interaction diagram depending