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Extended sandwich model for reinforced concrete slabs: Shear strength without transverse reinforcement

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ABSTRACT

In this paper, the description of the shear strength of orthogonally reinforced concrete slabs without transverse reinforcement by the newly developed extended sandwich model is presented. Based on a sandwich model, the slab element is subdivided into two cover elements and a core element, respectively; while the covers are subjected to in-plane forces only, the core has to resist to the transverse shear forces. Rotating, stress free cracks as well as tension stiffening effects according to the cracked membrane model are considered in the sandwich covers. Unlike to the covers, crack faces in the core are assumed to be able to transfer shear stresses by aggregate interlock. The fixed crack faces stand perpendicular to the slab plane, whereas the crack orientation relative to the slab plane is defined by the crack pattern of the covers. The influences of a deviation of the principal shear and moment direction from the direction of the in-plane reinforcement as well as of the slab thickness on the shear strength of slabs without transverse reinforcement are presented. Verifications against experimental data from beam and slab tests generally show a good agreement. Experimental evidence relating to size effects as well as deviations of the principal shear and moment direction from the in-plane reinforcement direction is confirmed.

1. Introduction

The shear strength of reinforced concrete slabs without transverse reinforcement is usually based on empirical and semi-empirical models. While current design codes regard the influence of the in-plane reinforcement ratios on the nominal failure shear stresses, only a few take into account the influence of the effective depth as well as a deviation of the principal shear direction from the in-plane reinforcement directions.

Early theoretical investigations [1] describe the internal mechanism of the brittle shear failures by introducing the so-called capacity of the concrete teeth and the tied concrete arch, that well correlate with experimental evidence. Based on comprehensive test series with reinforced concrete beams, Kani [2] has defined a region bounded by limiting values of the in-plane reinforcement ratio and the shear span inside which brittle shear failure occurs and outside which flexural strength is obtained. The size-dependence of the shear strength of reinforced concrete beams without transverse reinforcement has been captured since the sixties [3]. Recent developments of design models [4,5] have brought the shear strength response in correlation with the crack pattern as well as the stress of the in-plane reinforcement and the corresponding crack width. Also, tests on reinforced concrete slab specimens without transverse reinforcement [6,7] revealed a significant influence of the slab thickness as well as a deviation of the principal shear force direction from the in-plane reinforcement directions on the shear strength.

On the basis of a sandwich model [8] for slab elements subjected to transverse shear forces as well as flexural and twisting moments, a new mechanical model for cracked, orthogonally reinforced concrete slab elements with and without transverse reinforcement was developed, the extended sandwich model ESM of Jaeger [9–11]. Local effects like the punching shear strength are not treated and the beneficial effect of membrane forces is prudently neglected because of their sensitivity to unpredictable changes of the boundary conditions. After a review of the basics of the sandwich model, the present paper describes the shear strength of reinforced concrete slabs without transverse reinforcement with the new model. Parametric studies which demonstrate the influence of the slab thickness and the shear span ratio on the shear strength as well as comparisons with experiments conclude the paper. A comprehensive description of the flexural behavior is presented elsewhere [9,10].

2. Sandwich model

The dimensioning of orthogonally reinforced concrete slab elements can be based on a sandwich model [8]. The flexural moments m_x and m_y as well as the twisting moments $m_{xy} = m_{yx}$ as

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Nomenclature

A_s	cross-sectional area of flexural reinforcement
a	shear span
a_{sx}, a_{sy}	cross-sectional area per unit length of reinforcement in x- and y-direction
b	slab width
b_B, b_T	effective width of bottom and top cover
b_L	width of support plate
d	effective depth
d_a	maximum aggregate size
d_m	average effective depth
d_v	effective depth in shear
E_c	modulus of elasticity of concrete
E_s	modulus of elasticity of reinforcing steel
E_{sh}	modulus of strain hardening of reinforcing steel
F	applied jack force
f_c	effective concrete compressive strength
f_{cc}	cylinder compressive strength of concrete
f_{ct}	concrete tensile strength
f_{su}	ultimate strength of reinforcement
f_{sy}	yield strength of reinforcement
g	dead load of cantilever
h	slab thickness
M	flexural moment at support
$M_{u,\mathrm{exp}}$	observed ultimate flexural moment at support
$M_{u,\mathrm{ESM}}$	computed ultimate flexural moment at support
m_x, m_y	flexural moment per unit length in x- and y-direction
m_{xy}	twisting moment per unit length in x- and y-direction
m_u	flexural strength per unit length
m_{vR}	associated flexural moment to shear strength per unit length
n, t	coordinate
n_{1C}, t_{1C}	principal direction of applied stresses in sandwich core
n_x, n_y, n_{xy}	stress resultants per unit length in sandwich cover
s_r	diagonal crack spacing, crack spacing of secondary crack in sandwich cover
s_{r0}	diagonal crack spacing relative to principal shear direction
s_{rc}	crack spacing of primary crack in sandwich core
s_{rx}, s_{ry}	crack spacing in x- and y-direction
V	shear force at support
$V_{\mathrm{R,ESM}}$	failure criterion for ultimate shear force at support
$V_{\mathrm{u,exp}}$	observed ultimate shear force at support
$V_{\mathrm{u,ESM}}$	computed ultimate shear force at support
v_0	principal shear force per unit length
v_{1r}, v_{2r}	shear forces per unit length in $1r$- and $2r$-direction
v_R	shear strength per unit length
v_u	associated shear force to flexural strength per unit length
v_x, v_y	shear forces per unit length in x- and y-direction
w	deflection
w_{0r}	critical crack width in sandwich core with no aggregate interlock
w_r	crack width, crack width of secondary crack in sandwich cover
w_{rC}	crack width of primary crack in sandwich core
$w_{u,\mathrm{exp}}$	observed deflection at ultimate support moment
$w_{u,\mathrm{ESM}}$	computed deflection at ultimate support moment
x, y	coordinate
z	thickness of sandwich cover, coordinate
γ_c	concrete shear strain relative to principal shear direction
ε_1, ε_2	principal strain
ε_c	concrete strain
ε_{c0}	concrete strain at peak compressive stress
ε_{su}	ultimate steel strain
ε_z	strain in z-direction
θ_r	angle between x-axis and crack direction
ρ_n	effective geometrical in-plane reinforcement ratio in n-direction
ρ_x, ρ_y	geometrical reinforcement ratio in x- and y-direction
σ_c	concrete normal stress
σ_{c1C}, σ_{c2C}	principal concrete stress in sandwich core
σ_{cr}	normal stress at crack in sandwich core
σ_s	steel stress
τ_{1r}	shear stress at crack face in sandwich core
τ_c	concrete shear stress
τ_{c0r}	maximum shear stress at crack in sandwich core
τ_{cR}	ultimate nominal shear stress
τ_{cr}	shear stress at crack in sandwich core
$\tau_{u,\mathrm{exp}}$	observed nominal shear stress
$\tau_{u,\mathrm{ESM}}$	computed nominal shear stress
τ_{z0}	nominal shear stress relative to principal shear direction
φ_0	angle between x-axis and 0-axis (principal shear direction)
φ_{1C}	angle between 0-axis and principal direction of applied stresses in the core
φ_{1r}	angle between x-axis and principal strain direction in sandwich cover
ψ_C	principal stress direction relative to principal shear direction
ψ_{r0}	crack inclination in sandwich core relative to principal shear direction
ψ_{r1r}	crack inclination in core relative to principal strain direction of sandwich cover
0	principal shear direction
1, 2	principal direction of applied stress resultants in sandwich cover
$1r$, $2r$	principal strain direction in sandwich cover
Ø	reinforcing bar diameter
$Ø_x$, $Ø_y$	reinforcing bar diameter in x- and y-direction

shown in Fig. 1(a) can be resolved into equivalent in-plane forces acting on the bottom and top cover while the transverse shear forces v_x and v_y are assigned to the core of the sandwich, see Fig. 1(b). The core thickness that is equal to the effective shear depth, d_v, is given by the distance between the median planes of the bottom and top cover, where z_B and z_T denote the effective thickness of the sandwich covers. The transverse shear forces v_x and v_y correspond to a principal shear force

$$v_0 = \sqrt{v_x^2 + v_y^2} \tag{1}$$

which is transferred in a direction that encloses an angle

$$\varphi_0 = \tan^{-1}\left(\frac{v_y}{v_x}\right) \tag{2}$$

with the x-direction as shown in Fig. 1(c). Perpendicular to the direction of v_0 there is no shear transfer. Note that the shear force components v_x and v_y have no physical relevance.

Provided that the nominal shear stress

$$\tau_{z0} = v_0/d_v \tag{3}$$

relative to the principal shear direction does not exceed a certain value of about $f_{ct}/3$, the core is considered to be uncracked, where f_{ct} = tensile strength of concrete. Equal and opposite principal stres-