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Multi-dimensional moving least squares method applied to 3D elasticity problems

Kohei Sakihara [a], Hitoshi Matsubara [b,*], Taka-Aki Edo [b], Genki Yagawa [a,c]

[a] Center for Computational Mechanics Research, Toyo University, 2-36-5, Hakusan, Bunkyo-ku, Tokyo 112-0001, Japan
[b] Department of Civil Engineering and Architecture, University of the Ryukyus, 1, Senbaru, Nishihara-cho, Okinawa 903-0213, Japan
[c] School of Mechanical Engineering, World Class University(WCU), Program in Sungkyunkwan University, Suwon, Republic of Korea

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ABSTRACT

The aim of this study is to apply the "Multi-dimensional Moving Least Square method with Constraint condition (C-MultiMLS)" to the 3D elasticity problems in a weak form. We define the rotations, strains, and curvatures by using the C-MultiMLS, the degrees of freedom (DOFs) that should be finally solved in the simultaneous linear equation contain only displacement DOFs. This method is suitable also for the large-scale simulations. In this study, the accuracy of the proposed method in solving 3D elastic bodies is demonstrated in the solution of some numerical examples.

1. Introduction

Finite Element Method (FEM) has been adopted in a variety of general-purpose analysis software for solving engineering problems because of its easy implementation. Furthermore, in addition to engineering fields, this method has been applied in various other fields such as science, agriculture, and medical science, thus this method is of demonstrated value as a computational tool in these fields [1]. The most distinctive feature of the FEM is that it processes all setting of physical quantities such as boundary condition definitions, and other characteristics in an element which is a subdomain unit. Because of this, in case of discontinuous problems, some highly accurate re-meshing techniques are required. Thus, these techniques might be a large bottleneck in the application of the FEM [2,3].

Recently, various methods have been proposed for crack growth analyses without requiring re-meshing, which enable expansion of FEM applications. Typical examples of these methods are the manifold method (MM) [4], finite cover method (FCM) [5], and extended finite element method (XFEM) [6,7]. The MM, FCM and XFEM are defined both displacement field which is defined by mathematical approximation function and physical field which satisfies governing equation independently. These methods are also known as generalized finite element method (GFEM). However, the FEM is considered to have limited applicability to problems such as large deformation problems.

Under above-mentioned situation, numerical techniques such as mesh-less method and particle method, which do not require connectivity data between elements and nodes, have been studied intensively in order to overcome these bottlenecks [8–15]. These methods do not require creation or handling of meshes. Therefore, they stably deal with problems which include discontinuous surface or large deformation. These methods are rapidly expanding FEMs' applicability to various industrial fields. In case of the particle method, several discretization methods have been proposed, such as smoothed particle hydrodynamics (SPH) [16] or moving particle semi-implicit (MPS) [17]. However, study of their accuracy is considerably less compared with that of the FEM, and in future, it will be necessary to accelerate investigation on improving accuracy of solutions [18].

On the other hand, we have proposed a high-accuracy moving least squares method in which physical quantities such as rotations and strains can be defined for each particle [19]. In this method, the moving least squares method in one-dimensional error space by Lancaster and Salkauskas [20] is extended to multi-dimensional error space. We call this method "Multi-dimensional Moving Least Square method with Constraint condition (C-MultiMLS)." With this method, physical quantities such as rotations and strains can be directly introduced into physical field of each particle. Although the C-MultiMLS method is discussed in ref. [19], it is limited to implementation in the 2D problems and its accuracy evaluation, thus the study does not discuss its application to 3D elastic problems. In this paper, we will primarily discuss the methodology of applying the C-MultiMLS method to 3D elastic problems, and subsequently investigate the accuracy and features of results obtained by this method.

* Corresponding author. Tel.: +81 98 895 8652; fax: +81 98 895 8677.
E-mail address: matsbara@tec.u-ryukyu.ac.jp (H. Matsubara).

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2. Formularization of 3D elastic analysis with the C-MultiMLS method

2.1. Multi-dimensional Moving Least Squares Methods with Constraint condition (C-MultiMLS method)

It is well known that we can obtain high accuracy results because we can approximate high-order physical fields for each particle by using the traditional Moving Least Squares Method (MSLM). However, the approximation function is defined by just polynomial in this method, so that the unknown coefficients have no physical meanings. That is, we cannot directly evaluate the minimum square error of any physical values because only single value can be defined with the traditional MLSM. In general, physical values are often related to other physical ones, for example the rotation θ of elasticity in 2D is related to the displacements u_x and u_y, that is $\theta = (\partial u_y/\partial x - \partial u_x/\partial y)/2$. Therefore, there needs to be some numerical techniques that we can directly evaluate any physical quantities. Then, we here propose the newly error functional for the moving least-squares approximation, in which we can define the error space in multi-dimension which is shown in Fig. 1.

The C-MultiMLS method is formulated based on the Least Squares Interplant with Constraint Condition (CLS) method developed by Noguchi et al. [21]. The functional for the C-MultiMLS method is defined as following equation,

$$J = \sum_{j=1}^{N}\sum_{i=1}^{M} W(r_{ji}, h)\{u_{ij} - u_{ij}^{h}(\mathbf{x}; \alpha_1, \alpha_2, \ldots, \alpha_L)\}^2 \tag{1}$$

where J is the residual sum of squares, N is the number of neighboring particles, M is the total degrees of polynomial, $W(r_{ji},h)$ is the weighting function, r_{ji} is the distance between i and j particles, h is the radius of influence, u is the physical value, u^h is the arbitrary approximate function, and α is the unknown coefficients with physical meanings. In the weighting function $W(r_{ji},h)$, we choose the biquadratic B-spline function which is one of the radial basis functions [22] and is provided by

$$W(r_{ji}, h) = \frac{105}{16\pi h^3}\left(1 - 6\left(\frac{r_{ji}}{h}\right)^2 + 8\left(\frac{r_{ji}}{h}\right)^3 - 3\left(\frac{r_{ji}}{h}\right)^4\right). \tag{2}$$

This weighting function is one of the smoothing functions, which is zero outside the radius of influence (see Fig. 2). Note that the size-effect of the radius of influence is discussed in Section 3.2. By using Eq. (1), we can introduce some unknown parameters with physical meanings, for example rotations and strains to an arbitrary approximate function. (see Fig. 3).

2.2. Displacement field of a particle based on C-MultiMLS method

Let us focus on a particle i which is located at (x_i, y_i, z_i). The displacement fields $u(x, y, z)$, $v(x, y, z)$, and $w(x, y, z)$ can be approximated by the Taylor's second order expansion around the particle i as following equations,

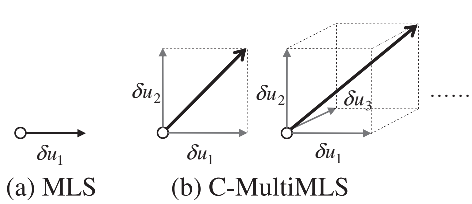

Fig. 1. Concept of C-MultiMLS.

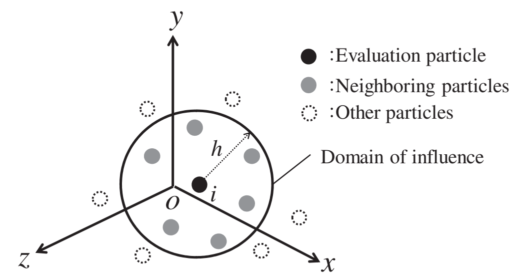

Fig. 2. Domain of influence from particle i (h: radius of influence).

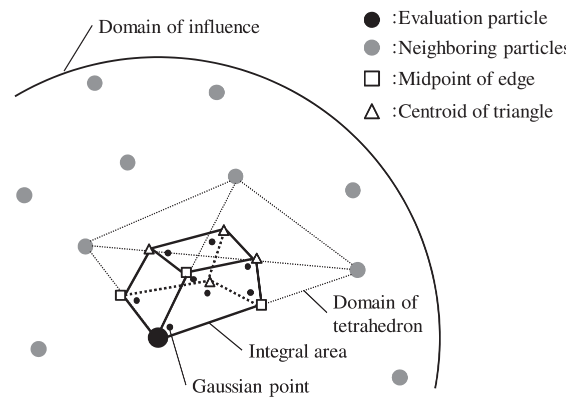

Fig. 3. Integration domain of particle.

$$u(x,y,z) \approx u_i + \frac{\partial u}{\partial x}\tilde{x} + \frac{\partial u}{\partial y}\tilde{y} + \frac{\partial u}{\partial z}\tilde{z} + \frac{1}{2}\left(\frac{\partial^2 u}{\partial x^2}\tilde{x}^2 + \frac{\partial^2 u}{\partial y^2}\tilde{y}^2 + \frac{\partial^2 u}{\partial z^2}\tilde{z}^2\right) + \frac{\partial^2 u}{\partial x\partial y}\tilde{x}\tilde{y} + \frac{\partial^2 u}{\partial y\partial z}\tilde{y}\tilde{z} + \frac{\partial^2 u}{\partial z\partial x}\tilde{z}\tilde{x} \tag{3}$$

$$v(x,y,z) \approx v_i + \frac{\partial v}{\partial x}\tilde{x} + \frac{\partial v}{\partial y}\tilde{y} + \frac{\partial v}{\partial z}\tilde{z} + \frac{1}{2}\left(\frac{\partial^2 v}{\partial x^2}\tilde{x}^2 + \frac{\partial^2 v}{\partial y^2}\tilde{y}^2 + \frac{\partial^2 v}{\partial z^2}\tilde{z}^2\right) + \frac{\partial^2 v}{\partial x\partial y}\tilde{x}\tilde{y} + \frac{\partial^2 v}{\partial y\partial z}\tilde{y}\tilde{z} + \frac{\partial^2 v}{\partial z\partial x}\tilde{z}\tilde{x} \tag{4}$$

$$w(x,y,z) \approx w_i + \frac{\partial w}{\partial x}\tilde{x} + \frac{\partial w}{\partial y}\tilde{y} + \frac{\partial w}{\partial z}\tilde{z} + \frac{1}{2}\left(\frac{\partial^2 w}{\partial x^2}\tilde{x}^2 + \frac{\partial^2 w}{\partial y^2}\tilde{y}^2 + \frac{\partial^2 w}{\partial z^2}\tilde{z}^2\right) + \frac{\partial^2 w}{\partial x\partial y}\tilde{x}\tilde{y} + \frac{\partial^2 w}{\partial y\partial z}\tilde{y}\tilde{z} + \frac{\partial^2 w}{\partial z\partial x}\tilde{z}\tilde{x} \tag{5}$$

where $\tilde{x} = x - x_i, \tilde{y} = y - y_i, \tilde{w} = w - w_i, \partial u/\partial x, \partial u/\partial y, \partial u/\partial z, \partial v/\partial x, \partial v/\partial y, \partial v/\partial z, \partial w/\partial x, \partial w/\partial y$ and $\partial w/\partial z$ are the first order derivative terms, and $\partial^2 u/\partial x^2$, $\partial^2 u/\partial y^2$, $\partial^2 u/\partial z^2$, $\partial^2 u/\partial x\partial y$, $\partial^2 u/\partial y\partial z$, $\partial^2 u/\partial z\partial x$, $\partial^2 v/\partial x^2$, $\partial^2 v/\partial y^2$, $\partial^2 v/\partial z^2$, $\partial^2 v/\partial x\partial y$, $\partial^2 v/\partial y\partial z$, $\partial^2 v/\partial z\partial x$, $\partial^2 w/\partial x^2$, $\partial^2 w/\partial y^2$, $\partial^2 w/\partial z^2$, $\partial^2 w/\partial x\partial y$, $\partial^2 w/\partial y\partial z$ and $\partial^2 w/\partial z\partial x$ are the second order derivative terms. The strains and rotations are defined based on elastic theory [1], which is provided by

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z} \tag{6}$$

$$\begin{aligned}
\gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}, \quad \theta_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\
\gamma_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \quad \theta_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\
\gamma_{zx} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \theta_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)
\end{aligned} \tag{7}$$