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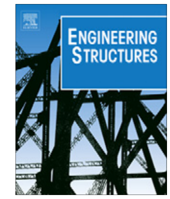

Dynamic analysis of sandwich beams with functionally graded core using a truly meshfree radial point interpolation method

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ABSTRACT

Transient responses and natural frequencies of sandwich beams with inhomogeneous functionally graded (FG) core are investigated. To serve this purpose, we propose a novel truly meshfree method in which the displacement field is approximated by the radial point interpolation method (RPIM) regardless of predefined mesh, and the domain integrals are evaluated by the so-called Cartesian transformation method (CTM) to obviate the need for a background cell. The effective properties of the FG core are obtained either by the rule of mixture or by the Mori–Tanaka micromechanics scheme, while the penalty technique is adopted to treat the material discontinuities at the interface between the core and the two face sheets. The accuracy and the efficiency of the present formulation are demonstrated by examining a series of numerical examples. The results are compared to those obtained by alternative methods, and excellent agreements are obtained.

1. Introduction

Over the past decades, most subjects on numerical modeling and simulation for engineering problems have been mainly carried out with the help of the finite element method (FEM) [1], the boundary element method (BEM) [2], and the finite difference method (FDM) [3]. As a well developed method, the FEM is now the most widely used numerical tool. Because of the highly mesh-dependent characteristics, the FEM itself, however, has many inherent shortcomings especially in the re-meshing processes when the elements become highly distortable. This may be a serious disadvantage in handling problems with high gradients, crack propagations, large deformations, shear-band strain localization, etc. In the contrary, due to the distinctive feature of being dependent only on nodes, meshless or meshfree methods (MMs) [4] have shown many advantages for handling such problems. Obviously, eliminating the aforementioned drawbacks encountered in the FEM is desirable. Several versions of MMs have been proposed so far with different formulations. The element-free Galerkin (EFG) method [5], meshless local Petrov–Galerkin (MLPG) method [6], radial point interpolation method (RPIM) [7], node-by-node meshless method (NBNM) [8], reproducing kernel particle method [9], meshless moving Kriging interpolation method (MK) [10,11], stable particle methods based on Lagrangian kernels [12], are just some representative examples of the MMs.

In contrast to the MMs, which are based on the local weak-form, e.g. the MLPG, those based on the global weak-form, e.g. the EFG, require an evaluation of domain integrals on the entire problem domain. Such domain integrals are frequently evaluated by the Gaussian quadrature (GQ) method using a background cell structure. By following this approach, an accurate evaluation of domain integrals especially in domains with complicated geometries often is a time-consuming task and the results are not completely satisfactory in many cases. Furthermore, such methods cannot be regarded as truly meshless. Here the term "truly" basically refers to methods in which neither a predefined mesh is used for constructing the field approximations nor a background cell is employed for the evaluation of domain integrals. Many efforts have been devoted to the elimination of the need for a background cell and some of them resulted in truly meshless methods, e.g. see [13–17]. The accuracy and computational costs of such meshless methods, however, are still a great concern and need further development. Another crucial component in the MMs that must be taken into account is the shape functions. The conventional moving least-square (MLS) approximation [5,6] is one of the widely used techniques, but its shape functions do not satisfy the Kronecker delta property. Therefore, the essential boundary conditions cannot be imposed directly. Direct collocation methods [18], Lagrange multipliers [5], penalty methods [19,20], modified variational principles [18], coupling with the traditional FEM [21–26], transformation

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method [27,28], are some of the commonly used special techniques developed for the imposition of the essential boundary conditions.

Composite materials and structures in general, and sandwich beams in particular, have been extensively used in different branches of engineering sciences because of their superior mechanical properties and their lightweight compared to other conventional engineering materials and structures. A sandwich beam commonly consists of a thick, light and low-modulus inner core covered by two thin, stiff and strong outer face sheets in order to obtain an efficient lightweight structure [29]. The core materials used in sandwich beams are essential to the overall performance of the structure. Although there are many cores offering certain beneficial advantages [29], but in many particular cases, developing new cores in order to best serve the specific purpose is an important task. As an example, a core made from functionally graded materials (FGMs) [30,31] which is studied in the present work, is potentially useful in many applications.

FGMs are inhomogeneous composite materials composed of different material constituents [30,31]. They differ from other composites in such a way that their properties vary gradually in specific directions. Sandwich beams with a FG core belong to a new generation of advanced structures and become increasingly attractive in many engineering applications. Certainly, a thorough understanding of their mechanical characteristics is firmly necessary. Apetre et al. [32] presented a combination of Fourier series and Galerkin method to investigate static contact and dynamic responses of such beams, Avila [33] proposed a failure mode criterion for accurately predicting the failure mechanisms, Kirugulige et al. [34] used both experimental and FEM techniques to investigate dynamic fracture behavior, Anderson [35] and Kashtalyan et al. [36] presented an analytical 3D elasticity solution method to study the effect of transverse loading, Amirani et al. [37] and Rahmani et al. [38], respectively, used the EFG and higher order sandwich panel theory for eigenvalue investigation. A 3D finite element model for low velocity impact was examined by Etemadi et al. [39]. For other problems concerning the applications of the FGMs, one can refer to [30,31,40], especially the works on dynamic responses of functionally graded plates and composite laminates [41–50].

The meshfree RPIM was introduced for the analysis of solid mechanics problems by Wang and Liu [7,51]. The method is further developed for solving many different problems, for instance, Biot's consolidation problems [52], static and dynamic analysis of 2D piezoelectric structures [53], 3D elasticity problems [54], flexural bending and free vibration of FGM plates [55], buckling of Mindlin plates [56], static analysis of laminated composite plates [57], strain smoothing techniques [58], geometrically nonlinear analysis of plates and shells [59], contact analysis of solids [60], and enrichment techniques for analysis of crack-tip fields [61], nonlinear transient heat conduction analysis of FGMs in the presence of heat sources [62], just to mention some application examples of the meshfree RPIM.

The primary objective of the present work is to propose a truly meshfree method with a high accuracy and efficiency in terms of the RPIM with the aid of a novel numerical integration technique for dynamic analysis of sandwich beams with a FG core. The advantages of this new approach are due to the desirable characteristics of the RPIM shape functions and the technique used for the evaluation of the domain integrals. Obviously, the Kronecker delta function property of the RPIM shape functions eliminates the necessity of special techniques used in the MLS method for the enforcement of the essential boundary conditions. Additionally, by utilizing a fast and efficient numerical integration technique, the so-called Cartesian transformation method (CTM) [63], the domain integrals are evaluated without any background mesh. In this way, a truly meshfree RPIM, also named as t-RPIM, is obtained with high accuracy and efficiency in comparison with conventional meshfree RPIM.

In the present work, the material discontinuities at the interface between the core and the face sheets are treated by using a penalty technique [4,37], whilst a homogenization for the material properties is performed by employing either the Mori–Tanaka micromechanics method [64,65] or the rule of mixture (ROM) [31,37,55]. The structure of the paper is outlined as follows. Some fundamental properties of the FG core are given in Section 2. A detailed description of the proposed method, including the construction of shape functions, elastodynamic formulation of the meshfree RPIM, and the CTM, is presented in Section 3. In Sections 4 and 5, some numerical examples for free and forced vibrations of the sandwich beam are presented using the proposed t-RPIM. Finally, some conclusions from the present work are drawn.

2. Functionally graded core

The first assumption made in this study is that the layers of the sandwich beam are perfectly bonded. The top surface of the FG core is assumed to be ceramic rich, whereas the bottom is metal rich. It is considered that the volume fraction of each constituent material follows a simple power law distribution, as follows [31,37,65]

$$V_c = \left(\frac{1}{2} + \frac{y}{t_c}\right)^n \quad \text{with } n \geqslant 0 \tag{1}$$

where V_c and $V_m = 1 - V_c$ are the volume fractions of the ceramic and the metal as indicated by subscripts c and m, respectively. In Eq. (1), y is the coordinate in the direction of the thickness of the beam, and t_c represents the thickness of the FG core. We denote by n the non-negative volume fraction exponent and its variation in general can alter the material properties significantly as depicted in Fig. 1. In particular, by assigning $n = 0$, we obtain a pure ceramic material, and by assigning $n = \infty$ a pure metal is obtained.

In order to find the effective properties at a point of the FG core, homogenization techniques such as the rule of mixture (ROM) [31,37] or the micromechanics (Micro) approaches [64,65] can be employed. In the present study, both homogenization techniques are used. The ROM is widely used because of its simplicity, and the effective property of the FG core at a point is given by

$$P_{eff} = P_m V_m + P_c V_c \tag{2}$$

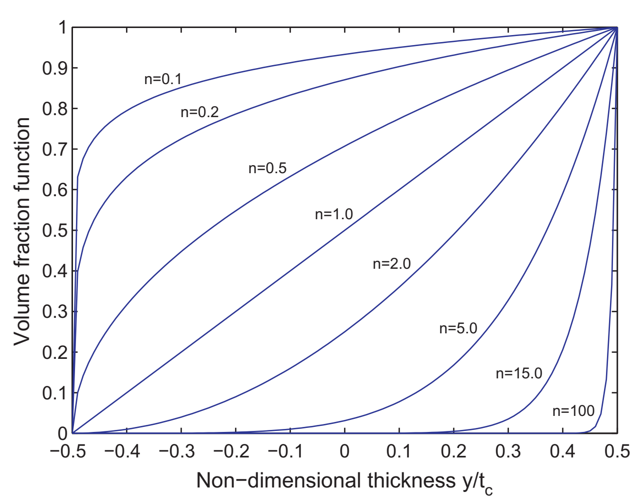

Fig. 1. Volume fraction versus the non-dimensional thickness for various non-negative volume fraction exponents of the FG core.