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Large eddy simulation of geometry sensitivity of magneotohydrodynamic turbulent flow in a rectangular duct

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ABSTRACT

Velocity distributions and pressure drops in liquid metal magnetohydrodynamic (MHD) duct flows are closely associated with the shape of the duct cross-section. In order to investigate the effects of the cross-sectional shape on MHD turbulent flows, liquid metal flow in two kinds of rectangular duct with a uniform magnetic field applied transversely to the flow are simulated using large eddy simulation with the coherent structure model. One duct has a normal rectangular cross-section (N-duct), while the other rectangular duct has a triangular protrusion pointing into the flow domain at the middle of the walls parallel to the magnetic fields (P-duct). Both duct walls are electrically insulating, and the inlet and outlet of the computational domain are set to be periodic. The flow Reynolds number is kept constant in this study, while the Hartmann numbers vary from 10 to 42.4. The numerical results show that the protrusion at the parallel walls promotes flow turbulence and compensates the effect of turbulence suppression due to the magnetic field. As the Hartmann number increases, the turbulent MHD flow transits to a single-sided turbulent flow and finally to a laminar flow. The protrusion at the side wall promotes the turbulence and delays the MHD turbulent-laminar transition as the Hartmann number increases. However, the skin friction coefficient is higher in P-duct than that in N-duct when the flow is turbulent. The protrusion cannot reduce the pressure drop in rectangular duct with insulating walls.

1. Introduction

Magneotohydrodynamic (MHD) flow in a duct is a fundamental problem in the study of the MHD industrial application, such as liquid metal blanket in nuclear fusion reactors, MHD pumps, and continuous casting process [1]. Velocity distributions and pressure drops in MHD duct flows are closely associated with the shape of the duct cross-section and the electrical conductivity of the duct [2,3]. The skin friction coefficient varies differently with the Hartmann number in laminar and turbulent magnetohydrodynamic duct flow. In laminar duct flow, the coefficient increases linearly with the Hartmann number. However, turbulent duct flow not only causes a reduction in the coefficient as the Hartmann number increases [4,5], but also promotes heat transfer, which is beneficial for the application of liquid metal blanket.

It is found that applying magnetic fields suppressed flow velocity fluctuation, which result in 'turbulent-laminar' transition [5–11]. Experimental and numerical results show that the transition depends on a parameter, *R*, defined as the ratio of Reynolds number to the Hartmann number ($R = Re/Ha$). The parameter *R* is the Reynolds number based on the Hartmann layer as the characteristic length. The transition threshold predicted by numerical simulation [5–11],8–11] is in the range of $R = 200 \sim 400$. As the *R* value decreases, flow in the duct core and the Hartmann layers near the wall perpendicular to the external magnetic field becomes laminar, while turbulence is sustained in the side layers near the wall parallel to the external magnetic field. The turbulent-laminar coexistence in the MHD flow in an insulating duct exists in a wide range of *R*. The single-sided turbulence pattern is critical, as it indicates the 'turbulent-laminar' transition [4,[5–11],10], and has only been observed in a long duct or pipe. A detailed review of the turbulent-laminar transition in magnetohydrodynamic duct, pipe, and channel flows can be found in the paper [12].

The turbulent-laminar transition is not only depended on the *R* value, but also geometry of the duct. Experimental results show that the protrusion in a square duct reduces the MHD pressure drop [13], as a result of the secondary flow generated by the protrusion. The structure that we investigate in this paper comes from the study in the paper [13]. But the wall in the experiment is electrically conducting, while the wall in the simulation is insulating.

The Coherent Structure Smagorinsky (CSM) and the dynamic Smagrinsky (DSM) subgrid-scale models have been found capable of

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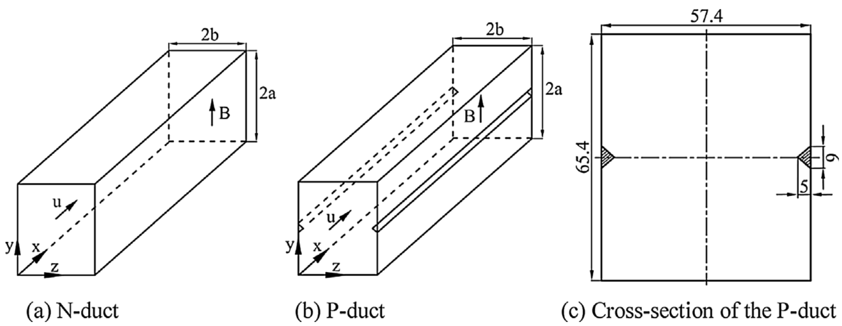

Fig. 1. Computational domain.

Table 1
Grid test Ha = 21.2, Re = 5681.

Duct	Grid ($N_x \times N_y \times N_z$)	Spatial resolution (Δx^+, Δy^+, Δz^+)	$C_f \times 10^3$
P-duct	128 × 120 × 80	32.0, 1.0 ~ 6.89, 0.84 ~ 5.40	6.919
	128 × 160 × 110	33.1, 0.21 ~ 12.3, 0.21 ~ 13.3	7.06
N-duct	128 × 90 × 90	32.9, 1.0 ~ 7.19, 0.88 ~ 4.80	7.04
	128 × 120 × 120	31.1, 0.19 ~ 18.0, 0.19 ~ 18.1	6.417
	256 × 120 × 120	15.55, 0.19 ~ 18.0, 0.19 ~ 18.1	6.10

predicting the turbulent-laminar transition associated with MHD duct flow [5,14,15]. Therefore, in this work the investigation of the transition is carried out using the CSM models in a rectangular duct with protrusion.

In this paper, we have investigated the effect of the protrusion on the MHD turbulent flow in a rectangular duct. The results are compared with those in the normal rectangular duct. It is concerned whether the protrusion in the duct can generate the secondary flow and decrease the skin friction. Because the side layer is the last zone that the turbulence of the liquid metal MHD flow in an insulating duct transforms to be laminar, we only consider the protrusion on the side walls in this paper.

2. Governing equations and numerical model

2.1. Governing equations

In industry application of MHD flow, the magnetic Reynolds number, $Re_m = U_0 D\sigma\mu_0$, is small. Here U_0 and D are the characteristic velocity and length, σ is the electric conductivity of the fluid, μ_0 is the magnetic permeability of the liquid metal. When $Re_{m<<}1$, the induced magnetic field can be ignored, or the magnetic field does not vary with time.

We consider an incompressible, liquid metal flow in an insulating rectangular duct subject to an uniform magnetic field. The governing equations with low magnetic Reynolds number are:

$$\nabla\cdot\boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} + \frac{1}{\rho}\mathbf{j}\times\mathbf{B}, \tag{2}$$

$$\mathbf{j} = \sigma(-\nabla\phi + \mathbf{u}\times\mathbf{B}), \tag{3}$$

$$\nabla^2\phi = \nabla\cdot(\mathbf{u}\times\mathbf{B}). \tag{4}$$

Where **u**, **j**, **B**, p,ν, ρ, σ, ϕ are velocity, electric current, applied magnetic field, pressure, kinematic viscosity, density, electrical conductivity and electric potential, respectively.

In MHD duct flow, the Reynolds number, the Stuart number (interaction parameter), and the Hartmann number are defined as $Re = U_m a/\nu$, $N = \sigma LB^2/\rho U_m$, $Ha = Ba\sqrt{\sigma/\rho\nu}$, where U_m, a are the average velocity and the half length of the duct parallel to the external magnetic field. The above three dimensionless parameters are also related by$Ha = \sqrt{Re\cdot N}$.

The large eddy simulation (LES) technique applies a spatial filtering to the unsteady Navier-Stokes Eq. (2), which yields [14,16]:

$$\frac{\partial\overline{\mathbf{u}}}{\partial t} + \overline{\mathbf{u}}\cdot\nabla\overline{\mathbf{u}} = -\frac{1}{\rho}\nabla\overline{p} + \nu\nabla^2\overline{\mathbf{u}} - \frac{\partial\tau_{ij}}{\partial x_j} + \frac{1}{\rho}\overline{\mathbf{j}}\times\overline{\mathbf{B}}. \tag{5}$$

where τ_{ij} is the sub-grid-scale stress tensor.

2.2. Coherent structure model

The sub-grid-scale model for LES used in this paper is Coherent Structure by [14,16], in which the stress tensor τ_{ij} is modeled as:

$$\tau_{ij} = -2(C\Delta)^2|\overline{S}|\overline{S}_{ij}, \tag{6}$$

$$|\overline{S}_{ij}| = \sqrt{2\overline{S}_{ij}\overline{S}_{ij}}, \tag{7}$$

$$C^2 = C_{CSM}\left|F_{CS}\right|^{3/2}F_\Omega,\ F_{CS} = \frac{Q}{E},\ F_\Omega = 1-F_{CS},\ C_{CSM} = \frac{1}{22} \tag{8}$$

Table 2
Parameters and computational details, Re = 5681.

Duct	Ha	R	Re_τ	Grid ($N_x \times N_y \times N_z$)	Spatial resolution (Δx^+, Δy^+, Δz^+)	Averaging time[a]
P-duct	10	568	348	128 × 160 × 110	33.50, 0.21 ~ 12.6, 0.21 ~ 13.6	9744
	21.2	268	341	128 × 160 × 110	33. 1, 0.21 ~ 12.3, 0. 21 ~ 13. 3	8292
	26.5	214	339	128 × 160 × 110	32.9, 0.20 ~ 12.3, 0.20 ~ 13.0	8173
	31	183	310	128 × 160 × 110	30.7, 0.19 ~ 11.4, 0.19 ~ 12.4	6289
	42.4	134	330	128 × 160 × 110	32.5, 0.20 ~ 12.0, 0.20 ~ 13.0	2138
N-duct	21.2	267	320	128 × 120 × 120	31.1, 0.19 ~ 18.0, 0.19 ~ 18.1	11069
	26.5	214	295	128 × 120 × 120	29.3, 0.18 ~ 17.1, 0.18 ~ 17.1	9826
	31	183	298	128 × 120 × 120	29.2, 0.18 ~ 17.0, 0.18 ~ 17.2	2503

[a] The averaging time is normalized by the time unit (a/U_m).