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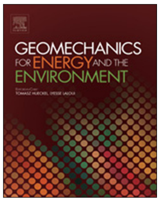

An analytical plane-strain solution for surface uplift due to pressurized reservoirs

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HIGHLIGHTS

- Exact poroelastic solutions for reservoir expansion and surface uplift.
- Conditions for when the 1D vertical solution is a good approximation.
- Verification of the exact solutions with numerical solutions.
- The usefulness of the 1D vertical approximation is demonstrated on a field case.

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ABSTRACT

In this paper, we present an analytical plane strain solution for surface uplift above pressurized reservoirs. The solution is based on a Fourier representation of the reservoir pressure. The plane strain model is developed in two stages: First, an exact solution is derived for the displacement field for the reservoir alone subjected to a periodic overpressure distribution of one wavelength. This one-layer model forms the basis for the analytical plane strain solution for a two-layer model — a pressurized reservoir with an overburden. We give an example where numerically computed uplift is quite accurately estimated by a simple 1D estimate, except for in the near well area. The plane-strain solution is well suited to study conditions for when the simple 1D approximation of the uplift is accurate. A condition for the accuracy of the simple 1D approximation is first derived for just the reservoir expanded by a periodic overpressure distribution of one wavelength, which corresponds to one term in a Fourier series. The 1D estimate is accurate for wavelengths larger than 2π times the reservoir thickness. Then, a condition is derived for when the 1D estimate is accurate for the two-layer model. We show that the wavelength of the overpressure distribution must be larger than 2π times the maximum of the reservoir thickness and the overburden thickness for the 1D approximation to be accurate. We demonstrate how uplift is computed from a Fourier decomposition of the reservoir overpressure. The resulting uplift is analysed in terms of Fourier coefficients, using the knowledge of how a single wavelength behaves. The analytical results for the displacement field and the uplift are tested by comparison with finite element simulations, and the match is excellent.

1. Introduction

The global emissions of CO_2 to the atmosphere are more than 35 Gt annually.[1] CO_2 is a greenhouse gas and the enormous emissions are one explanation for the observed global warming.[2] Subsurface storage of CO_2 in deep saline aquifers and depleted oil and gas reservoirs is considered a promising way to reduce the CO_2 emissions to the atmosphere.[3–5] The injection of large quantities of CO_2 into an aquifer or a reservoir leads to a build-up of pore fluid pressure. An increasing pore fluid will in turn lead to an expansion of the storage unit. Such expansion has been measured as surface uplift at the In Salah gas field using remote sensing techniques.[6] At In Salah, roughly 1 Mt CO_2 has been injected into a 20 m thick sandstone aquifer over 7 years.[7] The uplift is observed at a rate of roughly 5 mm/y around three injection wells.[7] A considerable amount of scientific interest has been devoted to understand surface uplift at In Salah and what it might imply for reservoir integrity.[7–19] In the same way as the onshore field In Salah, seabed uplift is expected for offshore

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CO_2 storage sites, although it is currently not straightforward to observe.

The surface uplift at In Salah has been modelled using numerical tools, like the mechanics simulator FLAC coupled to the two-phase flow simulator TOUGHREACT-II. The coupling of simulators for geomechanics and the multiphase flow is not a simple matter and the simulations are computer demanding.[20–24]

However, it is possible to estimate the uplift from just the pressure response in the reservoir using an analytical 1D linear poroelastic model.[7,25] Rutqvist[7] discuss the application of the 1D approximation at the In Salah site and he concludes that it overestimates the uplift near the wells. The overestimation is explained by the lateral variations of the fluid overpressure distribution in the reservoir, which are too large for a 1D model to be accurate. Such 1D estimates are very useful, since they can be applied directly to the pressure field from flow simulations, especially when the flow simulations can be performed decoupled from the geomechanics.[26] This method is extremely efficient when coupled with reduced order models for pressure increase from high volume injection of CO_2 into heterogeneous systems with large areal extent.[27] This simple method is therefore an attractive approach, however, its range of validity has been poorly quantified.

Analytical and semi-analytical models have been developed for the geomechanical response related to fluid injection into reservoirs and aquifers. Li et al.[28] present a semi-analytical model of a deformable reservoir coupled with immiscible two-phase flow (CO_2 and brine). The overburden is treated as a thin plate and the model computes axisymmetric flexural deformations due to a constant rate of CO_2 injection. The model is computationally light compared to the finite element simulations and it has been successfully applied to the In Salah field.

The full set of poroelastic equations can be computationally demanding to solve numerically. Therefore, it is customary to solve for the pore fluid pressure and the displacement field decoupled. One way to solve the poroelastic equations decoupled is by means of the fixed-stress split, which assures that a sequential solution of the two equations is unconditionally stable.[29,30] Another approach is developed by Andersen et al.[31,32] by using precomputed response functions.

A problem related to uplift by fluid injection is subsidence caused by fluid production. An early example of a subsidence model, based on poroelasticity and cylinder coordinates, was developed by Geertsma.[33,34] Selvadurai,[35] Kim and Selvadurai,[36] Selvadurai and Kim,[37] Niu et al.[38] have developed analytical models for different configurations of a reservoir and a caprock. Selvadurai and Kim[37] present analytical poroelastic solutions for a storage aquifer with a caprock, when there is a steady injection into a circular injection zone. The analytical solutions are given as integral representations and they are rather complicated expressions. The solutions were used to investigate how the radius and the depth of the planar injection region influence the surface displacement.

It should be mentioned that analytical solutions have limited applicability with respect to complicated geometries and complicated distributions of material properties. As with the analytical models cited in the paragraph above, we assume long sedimentary strata of homogeneous rocks. The challenge of upscaling heterogeneous rock units is outside the scope of this article.

In this paper, we develop a plane-strain solution of the displacement field for an overpressured reservoir with an overburden. We do this in terms of stationary analytical solutions of the poroelastic equations, when the reservoir overpressure is represented by a Fourier series. The reservoir layer is of infinite lateral extent with a periodic overpressure distribution. Overpressure is defined as the fluid pressure minus the initial fluid pressure, where the initial pressure is assumed hydrostatic. The expression for the displacement field gives the surface uplift. The solution is developed in two steps: The first is a one-layer model of just an overpressured reservoir. The next step, which builds on the first step, is a two-layer model of an overpressured reservoir with an overburden. These solutions for the displacement field are first developed for pressure as a single cosine-function, which is one term in a Fourier series. The one-layer and two-layer models are well suited to study the accuracy of a 1D estimate of uplift from a reservoir overpressure. We give answers in terms of wavelength. The poroelastic mechanical model is linear. Therefore, the full solution of the displacement field and the uplift is found as a superposition of solutions for the terms in the Fourier series representing the reservoir pressure. The use of plain strain assumption and Cartesian coordinates is different from the other approaches mentioned above, which are based on cylindrical coordinates. Another difference is that we study how a stationary fluid pressure controls the mechanical deformations, with respect to wavelengths.

This paper is organized as follows: The poroelastic assumptions are reviewed and an example is given where numerical computed uplift is compared with the 1D poroelastic uplift. The analytical model for the expansion of the reservoir is presented, before the analytical model for a reservoir layer with an overburden. The analytical results for uplift are first tested against numerical simulations for a single wavelength and then for pressure distributions represented by Fourier series.

2. Poroelasticity

The initial stress state is not modelled — it is taken as given. The difference from the initial stress state is modelled assuming linear poroelasticity.[39,40] Therefore, the full stress state is written as

$$\sigma_{ij} = \sigma_{ij}^{(0)} + \sigma_{ij}^{(1)} \tag{1}$$

where $\sigma_{ij}^{(0)}$ is the initial stress and where $\sigma_{ij}^{(1)}$ is the poroelastic stress change caused by fluid injection. The full stress state fulfils the equilibrium equations

$$\sigma_{ij,j} = \varrho g \delta_{iz} \tag{2}$$

where g is the constant of gravity, ϱ is the bulk rock density and δ_{ij} is the Kronecker delta

$$\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j. \end{cases} \tag{3}$$

The Einstein summation convention is applied in equilibrium equation (2), which means that there is summation over every pair of equal indices. The indices can have three values, 1, 2 and 3, which represent the three different spatial directions, respectively. An alternative to 1, 2 and 3 is x, y and z, respectively, with the exception that there are no summation over the x, y and z when they are used as indices. The initial stress state does also fulfil the equilibrium equations, which implies that the equilibrium equations in terms of the poroelastic stress difference becomes

$$\sigma_{ij,j}^{(1)} = 0 \tag{4}$$

where the right-hand-side is just zero. The right-hand-side of the equilibrium balance (4) could include a term representing buoyancy of supercritical CO_2 being less dense than brine. The following analysis is single phase and the eventual uplift from buoyancy is not accounted for, as commonly done with analytical models.[34,35,37] The effective stress τ_{ij} can be written as the sum of the initial effective stress $\tau_{ij}^{(0)}$ and the effective stress caused by changes in the fluid pressure $\tau_{ij}^{(1)}$, in a similar way to the full stress state

$$\tau_{ij} = \tau_{ij}^{(0)} + \tau_{ij}^{(1)}. \tag{5}$$