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Calculating local geomembrane strains from a single gravel particle with thin plate theory

H.M.G. Eldesouky, R.W.I. Brachman*

GeoEngineering Centre at Queen's–RMC, Queen's University, Kingston, ON K7L 4P5, Canada

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A new method is presented to calculate geomembrane strains induced by a single gravel particle (or grain) under vertical load from the deformed shape of the geomembrane for axisymmetric conditions. Past equations consider only vertical displacements of the geomembrane and neglect the contribution of radial displacements on strain and, consequently, underestimate the maximum strain. Axisymmetric large-strain-displacement relationships are used to relate radial strain to vertical and radial displacements. Vertical displacements are obtained from measurements of the deformed geomembrane from a physical experiment. Radial displacements do not need to be measured, but are related to tangential strain in the strain-displacement formulation. A linear elastic constitutive relationship is invoked and radial strains at the midsurface of the geomembrane from membrane elongation are solved for using Airy's stress function. Bending strains are obtained from the curvature of the deformed shape. Extreme fibre strains are the sum of the membrane and bending strains. Results from the new method match the maximum strain and pattern of strain when compared to large-displacement finite-element analysis. The new method is used to show that neglecting radial displacements underestimates the maximum strain by 25%, while neglecting radial displacements and bending strains underestimates the maximum strain by 60% (for a 2.5-mm-deep, Gaussian-shape indentation) and hence could affect selection of an appropriate geomembrane protection layer.

1. Introduction

Selection of an appropriate geomembrane protection layer requires assessment of the local tensile strains induced in the geomembrane from gravel particle (or grain) deformations (herein referred to as indentations) when subjected to construction and overburden loading. For HDPE geomembranes, tensile strains need to be limited to prevent brittle rupture and a corresponding increase in leakage in time (Abdelaal et al., 2014; Bannour et al., 2016; Brachman et al., 2017; Ewais et al., 2014; Müller, 2007; Rentz et al., 2017; Rowe et al. 2004, 2017; Seeger and Müller, 2003; Seeger and Müller, 1996; Yang et al., 2017). Gravel indentations are assessed from some sort of physical experiment, be it a more repeatable index test (Brachman and Sabir, 2013; Gallagher et al., 1999; Hornsey and Wishaw, 2012; Sabir and Brachman, 2012; Seeger and Müller, 2003; Seeger and Müller, 1996) with idealized gravel and/or subgrade conditions, or a more elaborate performance test (Abdelaal et al., 2014; Brachman et al., 2014; Brachman and Sabir, 2010; Dickinson and Brachman, 2008; Ewais et al., 2014; Joshi et al., 2017a, b; Rowe et al., 2013; Tognon et al., 2000; Zanzinger, 1999) conducted with site specific gravel and foundation soil at a certain pressure and for a certain time. Despite differences in testing, the permanent deformations of the geomembrane are recorded by a thin and soft metal sheet that is placed beneath the geomembrane. The metal sheet is retrieved after removal of the pressure and test materials and its deformed shape is measured. The geomembrane strain is then deduced from the deformed shape of the geomembrane. Key to this paper, is the premise that the magnitude and distribution of resulting strain depends on the assumptions made when calculating strain from the deformed shape.

One common approach is to take the measured deformed shape of the indentation, divide it into segments with width Δr and height Δz, and calculate the length of the deformed segment, L_f, using the Pythagorean theorem. In a cylindrical coordinate system (with radial direction r, tangential direction θ, and vertical direction z) in Fig. 1a, for segment i shown in Fig. 1b defined by Points a′ and b′, its deformed length would then be $L_{f\,i} = (\Delta r_i^2 + \Delta z_i^2)^{½}$. The elongation of each segment is calculated by assuming it reached that position by undergoing only vertical displacement, meaning Point a moved vertically to Point a′, and Point b moved vertically to Point b′. Thus, the initial length of segment i is $L_{o\,i} = \Delta r_i$ and its elongation is $\Delta L_i = L_{f\,i} - L_{o\,i}$. The

* Corresponding author.
E-mail addresses: hesham.eldesouky@queensu.ca (H.M.G. Eldesouky), brachman@queensu.ca (R.W.I. Brachman).

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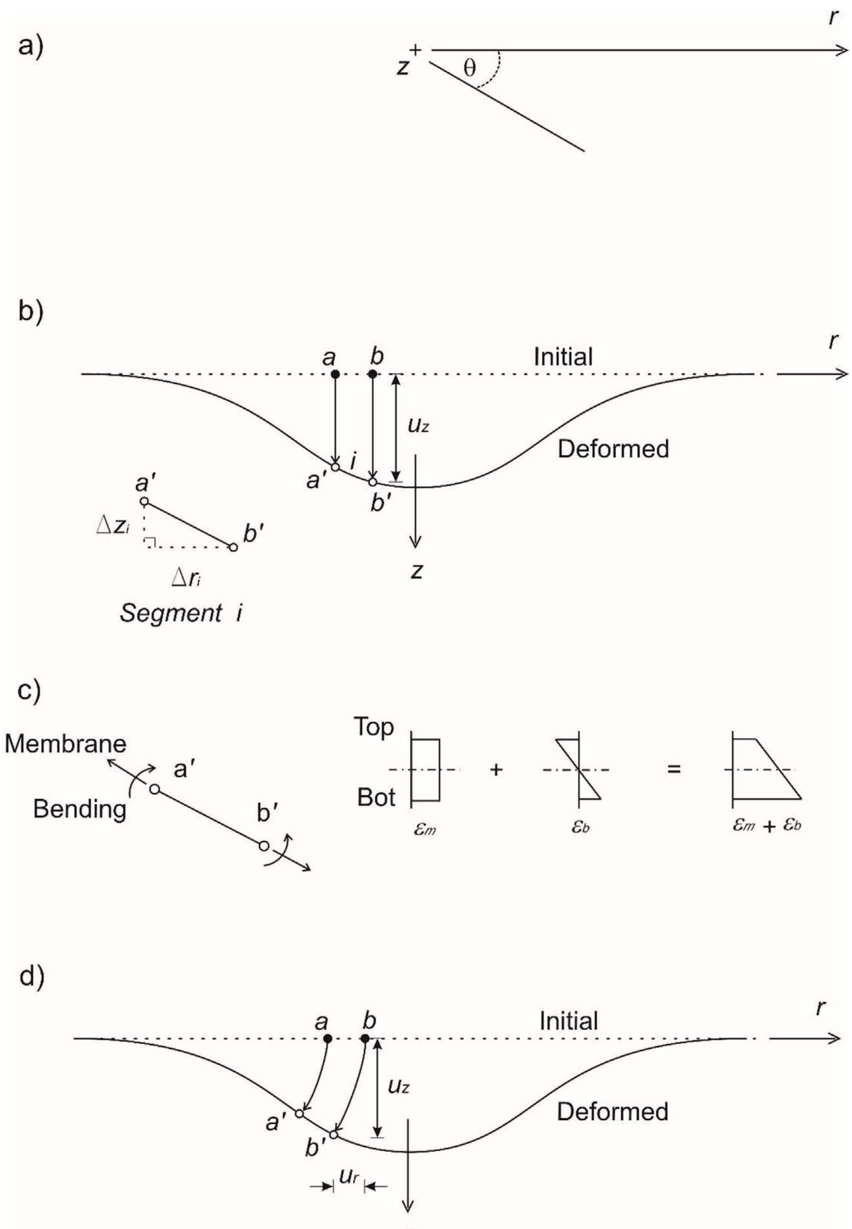

Fig. 1. Factors affecting calculation of geomembrane strain from deformed shape: a) plan view with definition of cylindrical co-ordinate system, b) segment elongation with no radial displacement, c) membrane and bending strain, and d) vertical and radial displacements.

elongation strain of segment i assuming only vertical displacement is then:

$$\varepsilon_i = \frac{\Delta L_i}{L_{o\,i}} = \frac{\sqrt{\Delta r_i^2 + \Delta z_i^2} - \Delta r_i}{\Delta r_i} \tag{1}$$

If that process is repeated for the entire deformed shape (i.e. for segments $i = 1$ to n) then the average elongation strain for the deformed shape is:

$$\varepsilon_{av} = \frac{\sum_{i=0}^{n} \Delta L_i}{b} \tag{2}$$

where b is the indentation width. Limiting the average elongation strain to 0.25% strain forms the basis of many acceptance criteria (e.g., BAM, 2015; EN-13719, 2016; Gallagher et al., 1999; Seeger and Müller, 2003; Seeger and Müller, 1996). Hornsey and Wishaw (2012) focussed on maximum elongation strain of the n segments obtained using Eq. (1), which is much larger than the average strain and was shown to be important for deep and irregularly shaped indentations from coarse angular gravel.

For the 5 mm deep axisymmetric indentation shown in Fig. 2a, the strain calculated using Eq. (1) (with Δr = 3 mm, as per EN 13719) is plotted in Fig. 2b. The largest segment elongation strain is 8.5% and occurs between 6 to 9 mm from the centre of the indentation while the average elongation over the indentation is 3.8%. Of note, is that the assumption of only vertical displacement (Fig. 1b) results in essentially zero elongation calculated at the centre of the indentation (r = 0), Fig. 2b. Decreasing Δr will improve the approximation to the actual strain profile. For example, Hornsey and Wishaw (2012) used a segment length of 1 mm. Tognon et al. (2000) treated the midsurface elongation of the geomembrane the same as illustrated in Fig. 1b and, using thin plate theory notation, called it membrane strain, ε_m. Using their finite difference formulation and a segment length of 0.5 mm results in a maximum membrane strain of 8.8% located at r = 7 mm and zero strain at the centre of the indentation, Fig. 2b.

Recognizing there is also extreme fibre strain from the curvature of the deformed shape, Tognon et al. (2000) added a bending component of strain ε_b, as defined in Fig. 1c and given by Kirchhoff–Love plate theory (Ventsel and Krauthammer, 2001):