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Probability and reliability analysis of pillar stability in South Africa

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ABSTRACT

A deterministic approach is frequently used in engineering design. In this quantitative design methodology, a safety factor, which is typically a strength-to-stress ratio, is derived as an index for the stability assessment of the engineering design. In underground coal mining applications such as pillar design, however, the inputs of pillar design are variables. This is widely overlooked in the deterministic approach. A probabilistic approach assessing the probability of failure or reliability of a system might be an alternative to the conventional quantitative methodology. This approach can incorporate the degree of uncertainty and deviations of variables and provide more versatile and reliable results. In this research, the reliability of case histories from stable and failed pillars of South Africa presented by Merwe and Mathey is examed. The updated Salamon and Munro strength formula (S-M formula) and Merwe and Mathey strength formula (M-M formula) are evaluated through a probabilistic approach. It is concluded that stable pillar cases have a reliability value greater than 0.83 while the reliability value of failed pillar cases are slightly larger than 0.50. There seems to be a positive relation between safety factor and reliability. The reliability of a pillar increases with pillar width but decreases with depth of cover, pillar height and entry width. The reliability analysis also confirms that M-M strength formula has a better distinction between the stable and failed pillar cases.

1. Introduction

In mining engineering, the deterministic method is widely employed for mine design. For instance, the safety factor is often introduced as a quantitative analysis methodology for pillar or slope stability design. The factor of safety is typically a ratio of strength to stress, thus an accurate estimation of strength and stress is required for a safe and good design [1,2]. Unfortunately, the physico-mechanical properties of engineering materials are always highly variable, uncertain or chaotic due to the complex in situ geological conditions. A limit of qualified testing techniques also contributes to unpredictability of the material strength. The anisotropy and heterogeneity of materials, for example, might lead to the strength scale effect or strength spatial variability. Likewise, the determination of stress remains challenging. This problem seems to be much more complicated in underground pillar design where the strength and stress are typically a function of entry width and pillar sizes such as pillar width and height. From a statistics point of view, these parameters are variables, strength and stress as a result should be expressed as variables. The conventional deterministic approach might be reliable. However, it uses the average or worst-case values of strength and stress rather than considering the uncertainty, discreteness or variability of these variables, which might lead to unsafe design in some cases. For instance, it is not uncommon in practice that some engineering structures failed with safety factors larger than 1 or stayed stable with safety factors less than 1 [3].

In civil and other geotechnical engineering, it is recognized that probabilistic approach might be an alternative for engineering design, which seems to be overlooked in the applications of coal mine design. As an alternative, the probabilistic approach is always associated with a progress of assessing the probabilistic failure of a system or reliability of an engineering construct. This approach might be considered as a meaningful methodology for coal mine pillar design by incorporating degree of uncertainty and deviations in variables. At present, reliability design via a probabilistic approach has been performed in practice in civil engineering, and some design standards have been formulated [4]. This approach receives greater attention in geotechnical engineering and surface mining. In underground coal mining, a number of applications have adopted probability and reliability analysis approach in the area of bolt support design [5,6]. But overall, the deterministic methodology is still the dominant approach in coal mine design.

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Monte Carlo methods are often used to solve problems having a probabilistic interpretation. Monte Carlo simulation is one of the most common means adopted in probability and reliability analysis to describe the variables of deterministic functions by using probability distribution or statistical characteristic values (mean values and standard deviations). With the development of computer technology in recent years, the implementation of Monte Carlo simulation becomes easier and more convenient, and its application in probabilistic design of civil and other engineering becomes more and more common and routine.

2. Methodology

2.1. Probabilistic approach

A factor of safety is used in pillar design to reflect the stability of pillars. It is often a strength-stress ratio which can be expressed as

$$SF = \frac{Strength}{Stress} = \frac{S_p}{\sigma_p} \tag{1}$$

where SF is safety factor; and S_p and S_σ the variables in the probabilistic approach.

Reliability can be defined as the ability or probability of a system or its components fulfilling the functions during a time period under specified conditions [7,8]. The reliability is usually given as:

$$R = 1 - P(failure) \tag{2}$$

where R is the reliability of a system or its components; and P (failure) the probability of failure of the system and given in Eq. (3):

$$P(failure) = P\{SF < 1\} = P\{S_p < \sigma_p\} \tag{3}$$

Like it is described in Fig. 1, strength and stress are generally independent and random variables which are assumed to follow a normal distribution. The failure region in Fig. 1 is defined as the probability of strength less than stress, which means the factor of safety is less than unity, or equivalently the probability of failure.

Let $Y = S_p - \sigma_p$, then variable Y will also follow a normal distribution. Then the probability of $Y > 0$ is equal to the reliability of a system, which can be given as:

$$R = P\{Y > 0\} = \int_0^\infty \frac{1}{\sqrt{2\pi}s_y} e^{-\frac{(y-\mu_y)^2}{2s_y^2}} dy \tag{4}$$

where μ_y and s_y are the mean value and standard deviation of variable Y respectively and are given as:

$$\mu_y = \mu_s - \mu_\sigma \tag{5}$$

$$s_y = \sqrt{s_s^2 + s_\sigma^2} \tag{6}$$

where μ_s and s_s are the mean value and standard deviation of variable S_p; and μ_σ and s_σ the value and standard deviation of variable σ_p, respectively.

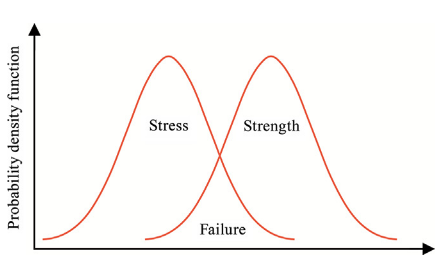

Fig. 1. Schematic of strength and stress probability density function.

2.2. Pillar strength

It is of difficulty to accurately determine the strength of a pillar in the field due to a lack of large-sized testing equipment. But it has been widely accepted that the specimen size has a significant effect on the strength of coal samples [9,10]. A common power equation is used for calculating pillar strength S_p, which can be expressed as a function of width-to-height ratio of a pillar and a constant:

$$S_p = k \cdot w^\alpha \cdot h^\beta \tag{7}$$

where w is pillar width; h the pillar height; α and β constants; and k a constant representing the strength of coal material.

By analyzing two databases of failed and stable pillar cases, Salamon and Munro derived an empirical pillar strength formula using maximum likelihood method [11]. In this method, it is assumed that the SF of failed pillars can be either larger or smaller than unity, which leads to a closer grouping of average SF of failed pillar cases around unity. Through an optimization process of maximum likelihood estimation from a 2013 updated stable and failed database, the constants in Eq. (7) can be determined, and Salamon and Munro coal pillar strength formula (S-M formula) can be written as [12]:

$$S_p = 6.61 \cdot \frac{w^{0.5}}{h^{0.7}} \tag{8}$$

Van der Merwe introduced the overlap reduction technique as an alternative formula for pillar strength calculation [13]. This technique was originally used in civil engineering to determine the reliability of structure performance. By arguing that a most-fit safety factor equation is based on the least overlap between the distributions of stable and failed pillar cases, the three constants are optimized and updated by fitting to the 2013 stable and failed database [12]. The Merwe and Mathey strength formula (M-M formula) can be given as:

$$S_p = 6.61 \cdot \frac{w^{0.5}}{h} \tag{9}$$

It should be noted that the updated coal pillar strength formulae for South African coal mines are based on 'normal' coal fields where weak floor conditions or weak coal areas are excluded, and the given cases in the database represent genuine pillar failures [14].

Bieniawski proposed a pillar strength formula by attempting direct strength tests on coal pillars with various sizes ranging from approximately 2 to 2 m [9]. The Bieniawski formula is shown in Eq. (10), which is related to width-to-height ratio of coal pillars and the compressive strength of coal.

$$S_p = S_i\left(0.64 + 0.36\frac{w}{h}\right) \tag{10}$$

where S_i is the coal compressive strength determined by field or laboratory-sized samples. Bieniawski formula gained its popularity in the US.

These empirical formulas are derived from back-analysis of pillar failures (Eqs. (8) and (9)) or from laboratory or in situ strength tests (Eq. (10)). An empirical approach might be deemed to be a lack of scientific explanation, thus it is necessary to understand the practical application of the strength formulas. In this research, a probabilistic methodology is employed to compare the applicability of different strength formulas. Since the databases used here are from South African coal mines, the comparisons of strength formulas are between S-M formula and M-M formula.