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A simple method for solving unidirectional methane gas flow in coal seam based on similarity solution

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ABSTRACT

The equation used to model the unidirectional flow of methane gas in coal seams is usually formulated as a nonlinear partial differential equation, which needs to be solved numerically with a computer program. Nevertheless, for people without access to the computer program, the conventional numerical method may be inconvenient. Thus, the objective here is to seek some method simpler than the conventional one for solving the flow problem. A commonly used model of the unidirectional methane gas flow is considered, where the methane adsorption is described by the Langmuir isotherm and the free gas is treated as real gas. By introducing the similarity solution, a simple method for solving the flow model is proposed, which can be done on a hand calculator. It is shown by two examples that the gas pressure profile obtained by the proposed method agrees well with the direct numerical solution of the flow model.

1. Introduction

As coal is porous in nature, significant amounts of methane can be retained in coal bed, and coalbed methane (CBM) is both a potentially valuable energy resource and a hazard in active coal mines [1–3]. Understanding the methane gas migration in coal seams is essential for both CBM recovering and coal mine gas control. But, the CBM flow equation is usually formulated as a nonlinear partial differential equation (PDE), which can hardly be solved by purely analytical techniques [4,5].

The unidirectional flow is one of the three basic patterns of methane gas migration in coal seams, while the other two are radial flow and spherical flow [6]. The unidirectional flow of methane gas is a kind of one-dimensional transient flow. It refers to the situation where the methane gas migrates towards a coal face, of which the height is equal to the thickness of the coal seam, as shown in Fig. 1.

For one-dimensional transient gas flow (without adsorption and desorption) in porous media, a traveling wave solution has been present by Hayek [7]. Because the CBM flow is usually accompanied by methane desorption [8,9], and the coal methane content is nonlinearly dependent on the pore gas pressure [10,11], the problem of CBM flow is more complex, and we have to solve it numerically using a computer program.

As is well known, there are many parameters controlling the methane gas flow in coal seams, such as gas pressure, coal seam permeability, adsorption coefficients, etc. While trying to investigate the effects of these parameters on the flow process, one needs to solve the flow model many times for different values of the parameters, and the computational cost rises rapidly as the parameter number increases. Moreover, and importantly, the conventional numerical analysis may be inconvenient for those, such as field engineers, field technicians, and so on, who might not have access to the computer program. Thus, the objective of this study is to seek a method for solving the flow model, which is simpler than the conventional numerical analysis, and is accurate enough, and can be done even without a computer. Of course, such a method can also be easily programmed to run efficiently on a computer.

2. Basic equation

While the methane adsorption is described by the Langmuir isotherm [12–14], and the free gas is treated as real gas [4,15], a commonly used model of the unidirectional methane gas flow can be written as [10,16]:

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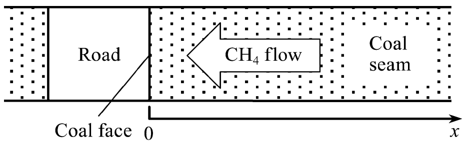

Fig. 1. Unidirectional methane gas flow in coal seam.

$$\frac{\partial P}{\partial t} = \frac{2M\sqrt{P}\left(1+b\sqrt{P}\right)^2}{1+\Phi(1+b\sqrt{P})^2/p_n} \cdot \frac{\partial^2 P}{\partial x^2} \tag{1}$$

where P is the square of the coal seam gas pressure, MPa^2, and $P = P(x, t) = p^2(x, t)$; p is the pore gas pressure, MPa; x is the spatial coordinate, m; t is the time coordinate, s; $M = \lambda/(ab)$, m^2/(MPa s); λ is the permeability coefficient of the coal seam, m^2/(MPa^2 s); a is the maximum adsorbed gas content, m^3/m^3; b is the equilibrium constant, MPa^{-1}; $\Phi = \phi/(ab)$, MPa; ϕ is the porosity; p_n is the normal gas pressure, here taking $p_n \approx 0.1$ MPa.

If the free gas is ignored, Eq. (1) reduces to:

$$\frac{\partial P}{\partial t} = 2M\sqrt{P}\left(1+b\sqrt{P}\right)^2 \frac{\partial^2 P}{\partial x^2} \tag{2}$$

For both Eqs. (1) and (2), the initial and boundary conditions are as follows:

$$\begin{cases} P(x,0) = P_0 = p_0^2 \\ P(0,t) = P_1 = p_n^2 \\ P(\infty,t) = P_0 = p_0^2 \end{cases} \tag{3}$$

where P_0 is the square of the original gas pressure in the coal seam, MPa^2; p_0 is the original gas pressure, MPa; P_1 is the square of the gas pressure at the coal face, and here we assume $P_1 = p_n^2 = 0.01$ MPa^2.

Obviously, Eqs. (1) and (2) with their initial and boundary conditions in Eq. (3) form two initial and boundary value problems (IBVP) of nonlinear PDE, which need to be solved numerically by using a computer program.

3. A simple method for solving the flow model

3.1. Similarity transformations

By introducing suitable similarity transformations, some PDEs can be reduced into ordinary differential equations (ODE) and the solutions of the ODEs are called the similarity solutions of the PDEs [17–21]. If the similarity solutions are known in advance, the solutions of the original PDEs can be easily obtained.

It is not too difficult to prove that both Eqs. (1) and (2) can be transformed into ODEs by introducing a similarity variable like $x/\sqrt{t}$. In order to reduce the number of the model parameters, and to make the initial and boundary conditions simpler, we define two similarity variables as follows:

$$\begin{cases} \eta = P/P_0 \\ \xi = x\sqrt{\frac{ab}{4\lambda P_0^{1/2} t}} \end{cases} \tag{4}$$

where η and ξ are similarity variables, dimensionless.

Substituting the similarity variables into Eqs. (1)–(3) separately leads to

$$-\xi\eta' = \frac{\sqrt{\eta}(1+\sigma\sqrt{\eta})^2}{1+10\Phi(1+\sigma\sqrt{\eta})^2}\eta'' \tag{5}$$

$$-\xi\eta' = \sqrt{\eta}\left(1+\sigma\sqrt{\eta}\right)^2\eta'' \tag{6}$$

and

$$\begin{cases} \eta(0) = \eta_0 \\ \eta(\infty) = 1 \end{cases} \tag{7}$$

where $\eta' = \frac{d\eta}{d\xi}$, $\eta'' = \frac{d^2\eta}{d\xi^2}$, $\sigma = b\sqrt{P_0} = bp_0$, and $\eta_0 = P_1/P_0 \in (0, 1)$.

So, the original two IBVPs have been transformed into two boundary value problems (BVP). Eqs. (5) and (6) are the so called similarity equations of Eqs. (1) and (2), separately. Obviously, the similarity equation is easier to be solved than its original equation. Moreover, it is worth mentioning that there are only two model parameters, i.e., σ and η_0, left in Eq. (6) and its boundary conditions. That means the solving of Eq. (6) only needs to be repeated for every (σ, η_0). So, it will be not too difficult to tabulate or graph the solutions of Eq. (6) in advance for further use to solve Eq. (2).

However, there are three model parameters, i.e., Φ, σ and η_0, left in Eq. (5) and its boundary conditions. So it will be difficult to tabulate or graph the solutions of Eq. (5) systematically in advance.

3.2. Proposed methods

Because the free gas is usually less than the adsorbed gas, Eq. (2) can be regarded as a rough approximation to Eq. (1), and Eq. (6) a rough approximation to Eq. (5). As the basis of following consideration, Eq. (6) is solved using the shooting method [22], and its numerical solution sets for a range of values of σ and η_0 are obtained as shown in Fig. 2. Here, only the solution sets for the cases of σ = 0.56, 1.8 and 5.8 are present because of the space restrictions.

Without loss of generality, suppose we are looking for the solution of Eq. (5) at $\xi = \xi_k$, with σ and η_0 known. Here, ξ_k is a certain value of ξ, which corresponds to some certain point of the space-time coordinate of (x, t). The basic ideas of the proposed method are as follows.

Firstly, we get the solution of Eq. (6) corresponding to the known σ and η_0 at $\xi = \xi_k$ from the solution sets as shown in Fig. 2, and denote this solution as $\eta_{f,k}$.

Secondly, we set

$$B = 1 + 10\Phi\left(1+\sigma\sqrt{\eta_{f,k}}\right)^2 \tag{8}$$

Thirdly, we rewrite Eq. (5) approximately as

$$-\xi\frac{d\hat{\eta}}{d\xi} = \frac{1}{B}\sqrt{\hat{\eta}}\left(1+\sigma\sqrt{\hat{\eta}}\right)^2\frac{d^2\hat{\eta}}{d\xi^2} \tag{9}$$

where $\hat{\eta}$ is an approximation of η.

Fourthly, we set

$$\hat{\xi} = \sqrt{B}\xi \tag{10}$$

and substitute it into Eq. (9), and get

$$-\hat{\xi}\hat{\eta}' = \sqrt{\hat{\eta}}\left(1+\sigma\sqrt{\hat{\eta}}\right)^2\hat{\eta}'' \tag{11}$$

where $\hat{\eta}' = \frac{d\hat{\eta}}{d\hat{\xi}}$ and $\hat{\eta}'' = \frac{d^2\hat{\eta}}{d\hat{\xi}^2}$.

The boundary conditions for Eq. (11) is

$$\begin{cases} \hat{\eta}(0) = \eta_0 \\ \hat{\eta}(\infty) = 1 \end{cases} \tag{12}$$

Obviously, Eq. (11) is identical in form to Eq. (6), and the boundary conditions in Eq. (12) are also identical to that in Eq. (7). So the solution of Eq. (11) can also be determined from the solution sets of Eq. (6) as shown in Fig. 2. We can take the solution of Eq. (11)

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