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Homogenization of the fluid-saturated piezoelectric porous media

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ABSTRACT

The paper is devoted to the homogenization of porous piezoelectric materials saturated by electrically inert fluid. The solid part of a representative volume element consists of the piezoelectric skeleton with embedded conductors. The pore fluid in the periodic structure can constitute a single connected domain, or an array of inclusions. Also the conducting parts are represented by several mutually separated connected domains, or by inclusions. Two of four possible arrangements are considered for upscaling by the homogenization method. The macroscopic model of the first type involves coefficients responsible for interactions between the electric field and the pore pressure, or the pore volume. For the second type, the electrodes can be used for controlling the electric field at the pore level, so that the deformation and the pore volume can be influenced locally. Effective constitutive coefficients are computed using characteristic responses of the microstructure. The two-scale modelling procedure is implemented numerically using the finite element method. The macroscopic strain and electric fields are used to reconstruct the corresponding local responses at the pore level. For validation of the models, these are compared with results obtained by direct numerical simulations of the heterogeneous structure; a good agreement is demonstrated, showing relevance of the two-scale numerical modelling approach.

1. Introduction

The piezoelectric effects which couple the mechanical deformation and the electrical field have been studied since the middle of eighteen century, however the piezoelectric materials became widespread during the World War I due to their use in resonators for detecting the acoustic sources produced by submarines using echolocation. After the World War II, apart of quartz, new types of the piezoelectric materials, such as barium titanite ($BaTiO_3$) and other synthesizes piezoceramic materials were developed with their dielectric constants much higher than those found in natural piezoelectric materials, such as quartz and some other minerals, or bone. Since then, the piezoelectric materials have found vast applications in electronics, mechatronics, and micro-system technology, being extensively used in the design of transducers, sensors and energy harvesters. Smart structures, such as microelectromechanical systems (MEMS) based on these materials allow for intelligent self-monitoring and self-control capabilities. Nowadays the piezoelectric sensor-actuator systems can be distributed continuously, being attached to the surface of other structural parts. Such an arrangement can be used e.g. in the aerospace industry to control vibrations, or acoustic radiation of thin flexible structures.

For modelling periodically heterogeneous media with piezoelectric components, classical upscaling techniques has been employed. Besides the micromechanics approaches including the Mori–Tanaka and self consistent upscaling schemes, (Ayuso et al., 2017), the classical periodic homogenization based on the formal two-scale asymptotic expansion method (Sanchez-Palencia, 1980; Cioranescu and Donato, 1999), or on the two-scale convergence (Allaire, 1992) and the periodic unfolding method (Cioranescu et al., 2008) has been used. Recently, the homogenization of thermoelectric and thermo-diffusive materials was treated in Fantoni et al. (2017) and Bacigalupo et al. (2016). Homogenization of the periodic composites consisting of piezoelectric matrix and elastic anisotropic inclusions accounting for bone cells was described in Miara et al. (2005); therein it has been suggested to exploit the piezoelectric effect in the design of a new type of bio-materials which should assist in bone healing and regeneration. Such possible application for piezoelectric materials in biomedical engineering is motivated by the electrochemical processes in biological tissues, which are coupled tightly with periodic mechanical loading assisted by the electric field. Performance of the tissue regeneration and remodelling may be enhanced by activated bio-piezo porous implants which can accelerate these processes undergoing at the microscopic level and related

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to the electro-mechanical transduction, cf. Sikavitsas et al. (2001); Wiesmann et al. (2001). The Suquet method of homogenization has been used to obtain analytic models of particulate and fibrous piezoelectric composites (Iyer and Venkatesh, 2014). Apart of the homogenization of heterogeneous piezoelectric media, an asymptotic analysis has been applied to derive higher order models of piezoelectric rods and beams, starting from the 3D piezoelectricity problem (Viaño et al., 2016).

In Rohan and Miara (2006), the shape sensitivity formulae were derived for a class of 2D microstructures comprising one piezoelectric and one arbitrary elastic material, whereby the shape of the interface between the two materials was parameterized. As a challenge for the material design, the numerical tests have shown how a suitable geometry of the interface can amplify some of the homogenized coefficients, namely the third-order tensors associated with the electromechanical coupling. Sensitivity of the effective medium properties to the microstructure properties were also reported in Koutsawa et al. (2010).

Besides the periodic homogenization, in Ayuso et al. (2017), the Mori–Tanaka and the self-consistent schemes were used for upscaling the drained porous piezoelectric materials. Concerning the fluid saturated porous piezoelectric media, the asymptotic method has been applied in Telega and Wojnar (2000) to derive macroscopic constitutive laws accounting for the fluid-structure interaction at the pore level, whereby a simplified model of electrolytes was considered. In the context of the bone tissue biomechanics, the macroscopic influence of piezoelectric effects observable in dried bone was studied in Lemaire et al. (2011) using the homogenization approach.

Propagation of electroacoustic waves in an reinforced piezoelectric medium was treated in Levin et al. (2002). The low frequency acoustic wave propagation in the porous piezoelectric materials has been subject of several works (Vashishth and Gupta, 2009; Sharma, 2010; Vashishth and Gupta, 2011; Besombes et al., 0000). In these papers, the modelling is based on the Biot theory of porous media elaborated within the phenomenological approach, therefore, influences of specific microstructures on the wave dispersion have not been studied yet.

This paper is focused on the derivation of the effective material coefficients of the fluid-saturated porous media with the piezoelectric skeleton using the homogenization framework. A related topic was treated recently in Iyer and Venkatesh (2014), where a special type of piezoelectric anisotropic composite materials was studied using numerical and analytical methods. Although the porosity influence was examined and the figures of merit related to the hydrostatic strain coefficient were also investigated, we pursue another homogenization approach which is based on the periodic homogenization of the static fluid-structure interaction, as reported in Rohan et al. (2015) in the context of the hierarchical porous poroelastic media, cf. Rohan and Lukeš (2015). We assume a quasistatic loading, such that inertia and viscosity related effects can be neglected. As the consequence, in any connected porosity, a unique pressure is established which satisfies the equilibrium. Using the homogenization of the fluid-structure interaction problem at the microscopic scale, we obtain macroscopic models of the upscaled piezo-poroelastic medium for different periodic microstructures; one connected porosity, or an array of fluid filled inclusions is combined with piezoelectric skeleton which can contain mutually separated conductors (metallic parts). We consider two different situations: (1) the conductors are distributed as a periodic arrays of mutually separated inclusions, or (2) the conducting pars constitute two or more electrodes such that each of these electrodes presents a connected porous structure. In the second case, different electric potential is prescribed to different electrodes, so that electric fields induced in the microstructure can be controlled.

The paper is organized as follows. In Section 2, different microscopic configurations of the periodic porous piezoelectric medium are defined and the model equations are introduced, yielding the weak formulation. The homogenization of the static fluid-structure interaction is reported in Sections 3 and 4, for the two above mentioned designs of the conducting parts. In both these sections, the local problems for the characteristic responses of the representative period cell are derived and the homogenized effective material coefficients are obtained. These are involved in the macroscopic equations governing behaviour of the upscaled poro-piezoelectric medium. Using the characteristic responses and the macroscopic fields, the displacement, pressure and electric fields can be reconstructed at the microscopic level, as reported in Section 6. Finally, in Section 7, we present numerical illustration of the derived macroscopic models. For validation of these models, direct numerical simulation of the heterogeneous media are compared with the responses computed using the homogenized problems. Some technical supporting material is explained in the Appendix.

Some basic notations. In the paper, the mathematical models are formulated in a Cartesian framework of reference $\mathcal{R}(O; \boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$ where O is the origin of the space and $(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3)$ is a orthonormal basis for this space. The coordinates of a point M are specified by $x = (x_1, x_2, x_3)$ in $\mathcal{R}$. The boldface notation for vectors, $\boldsymbol{a} = (a_i)$, and for tensors, $\boldsymbol{b} = (b_{ij})$, is used. The following special notation is used for the electric field $\vec{E}$, and the electric displacement vector $\vec{D}$. Furthermore, a special notation is introduced for the 3rd order tensors associated with piezoelectric coupling, $\underline{\boldsymbol{G}}^H$, $\underline{\boldsymbol{g}}$. The gradient and divergence operators are respectively denoted by ∇ and $\nabla\cdot$. When these operators have a subscript which is space variable, it is for indicating that the operator acts relatively at this space variable, for instance $\nabla_x = (\partial_i^x)$. The small strain tensor is denoted by $\boldsymbol{e}(\boldsymbol{u}^\varepsilon) = (\nabla \boldsymbol{u}^\varepsilon + (\nabla \boldsymbol{u}^\varepsilon)^T)/2$. The symbol dot '$\cdot$' denotes the scalar product between two vectors and the symbol colon ':' stands for scalar (inner) product of two second-order tensors. Throughout the paper, x denotes the global ("macroscopic") coordinates, while the "local" coordinates y describe positions within the representative unit cell $Y \subset \mathbb{R}^3$ where $\mathbb{R}$ is the set of real numbers. By dV (or dV_x) and dV_y we denote the elementary volume elements associated with coordinates x and y, respectively, while dV_{xy} is the elementary volume in a cross-product domain $\Omega \times Y$. Accordingly, elementary surfaces are designated by dS, dS_x and dS_y. By $\fint Y_D = |Y|^{-1} \int_D$ with $Y_d \subset \overline{Y}$ we denote the local average. The Lebesgue spaces of 2nd-power integrable functions on a domain D is denoted by $L^2(D)$, the Sobolev space $\boldsymbol{W}^{1,2}(D)$ of the square integrable vector-valued functions on D including the 1st order generalized derivative, is abbreviated by $\mathbf{H}^1(D)$. The unit normal vector outward to domain D_s is denoted by $\boldsymbol{n}^{[s]}$.

2. Microscopic model of porous piezoelectric media

There are typically two characteristic lengths: ℓ describes the heterogeneity size and L is the relevant macroscopic size. The ration $\varepsilon = \ell/L$ is called the scale parameter. As usually, we consider material properties of the heterogeneous medium oscillating with period ℓ relative to the spacial position. The asymptotic method of homogenization is based on the asymptotic analysis of the mathematical model for $\varepsilon \to 0$.

2.1. Periodic microstructure

The medium is generated by copies of the representative volume element (RVE) $Z^\varepsilon \subset \mathbb{R}^3$ as a periodic lattice, so that εa_k is the lattice period in the k-the coordinate direction. For the "real size" RVE, we introduce its rescaled copy $Y = \varepsilon^{-1} Z^\varepsilon$ which is called the rescaled elementary periodic cell Y defined by

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