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New insight on physical meaning of fracture criteria for growing cracks

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ABSTRACT

Because plastic unloading is inevitable during crack growth in elastic plastic materials, the J integral is not applicable anymore, and several path-independent integrals are thus proposed by different authors to overcome this limitation. In this paper, we start from the crack tip energy flux. We first derive two important formulas and then find that these path-independent integrals are in fact identical to each other and consistent with the crack tip energy flux. That is, the crack tip energy flux is more essential for growing cracks in elastic plastic materials. In addition, a force such as crack tip parameter, which is conjugated to crack tip velocity, is proposed by us on the perspective that crack tip energy flux can be considered as the power dissipated by crack tip due to crack growth. This parameter is proved to be the absolute value of the projection of the crack tip configurational force vector into the direction of the nominal crack growth direction, and thus, this parameter is the thermodynamic crack driving force. Moreover, this parameter can represent crack tip stress and displacement fields of growing cracks, and thus, a fracture criterion based on this parameter is proposed. The critical value of this parameter corresponds to the rate of energy dissipated in the fracture process zone per unit crack extension.

1. Introduction

Since the pioneering works of Eshelby (1956) and Rice (1968), the J integral has achieved great success as a fracture characterizing parameter for nonlinear materials. However, the J integral is based on the deformation theory of plasticity and ceases to be a valid parameter for growing cracks in elastic plastic materials because plastic unloading always exists.

To overcome this limitation, several integral-type crack tip parameters are proposed from completely different starting points. The path-independent integral $\hat{J}$ proposed by Kishimoto et al. (1980) was among the first attempts to introduce the path-independent integral, which is not restricted by any specific material behavior. Kishimoto et al derived the formula of $\hat{J}$ from modified energy balance during crack growth by considering the concept of fracture process zone (FPZ) proposed by Broberg (1971, 1975), in which usual continuum mechanics is not applicable.

In addition, Atluri and his collaborators (Atluri, 1982; Atluri and Nishioka, 1984; Brust et al, 1985, 1986) changed the strain energy density defined in the J integral into accumulated stress working density, which is suitable for the increment theory of plasticity, and then defined a path-independent integral, namely T^* integral. They found that T^* and the J integral curves almost coincide when crack initiates; as cracks further propagate, more and more differences are observed. When steady state is reached, T^* becomes a constant while J continues to rise.

Recently, the surface-forming energy release rate $G_s$was proposed by Xiao et al. (2015, 2017) according to power balance during crack propagation. For the reason that it is in the rate form, this concept can capture the plastic unloading of elastic plastic materials and thus is applicable to crack growth in elastic plastic materials. Besides, Xiao et al. (2015) proved that $G_s$is also a path-independent integral.

In nature, all these abovementioned integrals are revisions of the J integral. Furthermore, all of them are argued by their authors that they can be used as the elastic plastic fracture criterion. Some confusion may arise about how to choose these crack tip integrals or, more essentially, whether these crack tip integrals are a crack driving force or not. To tackle this problem, we must first explore the nature of crack growth fracture criterion; in other words, the physical meaning of crack growth criterion must be understood. This is the main purpose of this study.

In recent years, particular attention has been given to the application of configurational force approach (Gurtin, 1995, 2000; Gurtin and Podio Guidugli, 1996) to fracture mechanics. We only focus on the work of Simha et al. (2003, 2005, 2008) and Kolednik et al. (2014) because the near-tip J integral proposed by

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Simha et al. (2008). The near-tip J integral was directly derived from Clausius-Duhem inequality (second law of thermodynamics) of the body containing the crack tip. The near-tip J integral was shown to be the thermodynamic crack driving force and be consistent with the crack tip configurational force. This parameter may provide a new perspective to review the crack tip integrals.

This paper is organized as follows: In Section 2, two important formulas Eqs. (19) and ((26)) are obtained. According to the two formulas, we find that surface forming energy release rate G_s, $\hat{J}$ integral, and T^* integral are in fact equivalent to each other and consistent with the crack tip energy flux. In other words, the crack tip energy flux is more essential. In Section 3, a parameter $F_T$is proposed by us because the crack tip energy flux can be considered as the power dissipated by crack tip due to crack growth. Furthermore, this parameter is shown to be identical to the above path-independent integrals. Because it is the force term conjugate to the crack tip velocity, this parameter can be regarded as the crack driving force. Besides, this parameter is proved to be minus the projection of the crack tip configurational force vector into the direction of the nominal crack growth direction. That is, this parameter is associated with the near-tip J integral proposed by Simha et al. (2008). In addition, the physical meaning of the J integral for growing cracks in elastic plastic materials is also discussed. In Section 4, we find that $F_T$can represent crack tip stress and displacement fields of growing cracks by the equivalent domain integral method. Furthermore, the F_T-based fracture criterion is thus proposed. Main conclusions are summarized in Section 5.

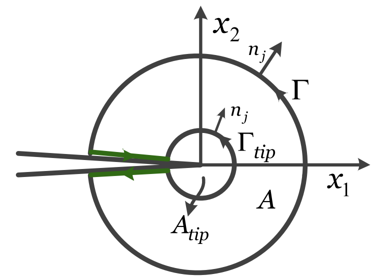

Fig. 1. A finite and an infinitesimal contour surrounding the crack tip.

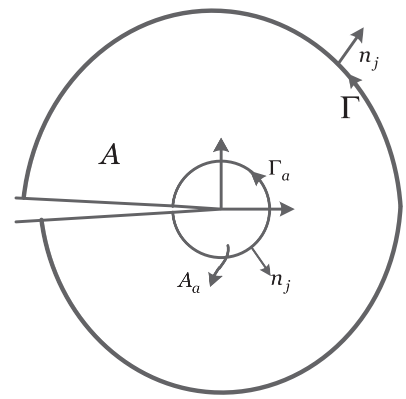

Fig. 2. Crack tip conventions.

2. The unified crack tip integral-crack tip energy flux

As cracks propagate in elastic plastic materials, plastic unloading will occur behind crack tips, and then, the J integral-based criterion is no longer applicable. Several integrals have already been proposed to tackle this problem. They are $\hat{J}$integral based on FPZ (Kishimoto et al, 1980), T^* integral from the incremental theory of plasticity (Atluri et al, 1982, 1984, Brust et al, 1985), and the surface forming energy release rate $G_s$derived from the power balance of crack extension (Xiao et al, 2015, 2017). In this section, we show that all these integrals are consistent with the crack tip energy flux, which can be obtained from the first law of thermodynamics for the cracked body.

Consider an area A within a curve Γ surrounding the crack tip, which starts from a point on the lower crack surface and ends at a point on the upper crack surface.$G_s$was defined by Xiao et al. (2015) as follows:

$$G_s\dot{a} = \int_\Gamma \sigma_{ij} n_j \dot{u}_i dA - \int_A \sigma_{ij}\dot{\varepsilon}_{ij} dA \tag{1}$$

where a is the crack length; $n_j$represents the unit external normal vector of Γ; and σ_{ij}, ε_{ij}, and $u_i$are the stress, strain, and displacement components, respectively. $\dot{a}$ represents the temporal derivative of the parameter a.

The definition of G_s has used several assumptions:

(1) Small strain conditions: $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$;
(2) Crack surfaces within the contour Γ are assumed to be traction free;
(3) Body forces are ignored;
(4) The crack propagation is assumed to be quasi-static with the kinetic energy ignored.

Now consider a small contour Γ_{tip} (the area is denoted A_{tip}) in the contour Γ as in Fig. 1.

From the abovementioned assumptions, we can write the equilibrium equation as follows:

$$\sigma_{ij,j} = 0 \tag{2}$$

By multiplying both sides of the equilibrium equation by $\dot{u}_i$, we have

$$\int_{A-A_{tip}} \sigma_{ij,j}\dot{u}_i dA = \int_{A-A_{tip}} (\sigma_{ij}\dot{u}_i)_{,j} dA - \int_{A-A_{tip}} \sigma_{ij}\dot{u}_{i,j} dA = 0 \tag{3}$$

According to the Gauss theorem, Eq. (3) can be rearranged into

$$\int_{\Gamma_{tip}} \sigma_{ij} n_j \dot{u}_i d\Gamma = \int_\Gamma \sigma_{ij} n_j \dot{u}_i d\Gamma - \int_{A-A_{tip}} \sigma_{ij}\dot{\varepsilon}_{ij} dA \tag{4}$$

Eq. (4) is identical to the formula of $\hat{J}\dot{a}$ according to Kishimoto et al. (1980). In their paper, the area $A_{tip}$was taken as the region of FPZ, in which usual continuum mechanics does not work. From Broberg (1971, 1975), FPZ is quite small, and the singular crack tip is essentially an ideal FPZ. Thus, $\Gamma_{tip} \to 0$.

For stationary cracks, the term $\sigma_{ij}\dot{\varepsilon}_{ij}$in K or HRR fields has the order of r^{-1}. The crack tip fields of growing cracks in elastic plastic materials have a less singularity (Rice et al., 1980). Thus

$$\lim_{A_{tip}\to 0} \int_{A_{tip}} \sigma_{ij}\dot{\varepsilon}_{ij} dA = 0 \tag{5}$$

Therefore, Eq. (4) can be rewritten as follows:

$$\hat{J}\dot{a} = \int_{\Gamma_{tip}} \sigma_{ij} n_j \dot{u}_i d\Gamma = G_s\dot{a} \tag{6}$$

Namely, G_s and $\hat{J}$ are in nature identical. By closely following the method of Nakamura et al. (1985) and Moran et al. (1987a, b), we can get the crack tip energy flux.

Consider a two-dimensional cracked body in a rectangular Cartesian coordinate system (Fig. 2). The crack extends along the $x_1$axis with instantaneous speed $\dot{a}$. At a given instant, this body area A is bounded by a fixed contour Γ. Picking a small contour $\Gamma_a$surrounding the crack tip, this contour is fixed in size and orientation with respect to the crack tip and is translating with the crack tip. The area bounded by $\Gamma_a$is denoted as A_a. The two contours are connected by the crack surfaces, and the crack surfaces are assumed to be traction free. Again, the body forces are ignored,