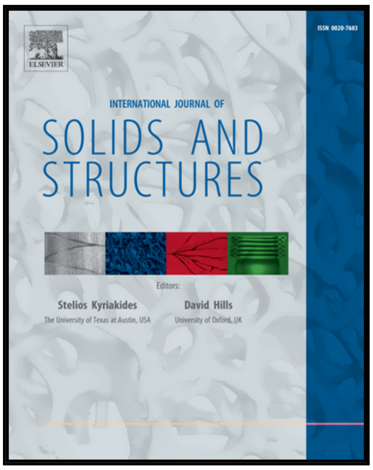

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Acoustic wave polarization and energy flow in periodic beam lattice materials

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Abstract

The free propagation of acoustic plane waves through cellular periodic materials is generally accompanied by a flow of mechanical energy across the adjacent cells. The paper focuses on the energy transport related to dispersive waves propagating through non-dissipative microstructured materials. The generic microstructure of the periodic cell is described by a beam lattice model, suitably reduced to the minimal space of dynamic degrees-of-freedom. The linear eigenproblem governing the wave propagation is stated and the complete eigensolution is considered to study both the real-valued dispersion functions and the complex-valued waveforms of the propagating elastic waves. First, a complete family of nondimensional quantities (*polarization factors*) is proposed to quantify the linear polarization or quasi-polarization, according to a proper energetic criterion. Second, a vector variable related to the periodic cell is introduced to assess the directional flux of mechanical energy, in analogy to the Umov-Poynting vector related to the material point in solid mechanics. The physical-mathematical relation between the energy flux and the velocity of the energy transport is recognized. The formal equivalence between the energy and the group velocity is pointed out, according to the mechanical assumptions. Finally, all the theoretical developments are successfully applied to the prototypical beam lattice material characterized by a periodic tetrachiral microstructure. As case study, the tetrachiral material offers interesting examples of perfect and nearly-perfect linear polarization. Furthermore, the nonlinear dependence of the energy fluxes on the elastic waveforms is discussed with respect to the acoustic and optical surfaces featuring the energy spectrum of the material. As final remark, the occurrence of negative refraction phenomena is found to characterize the high-frequency optical surface of the frequency spectrum.

Keywords: Acoustic waves, polarization, energy flux, Umov-Poynting vector, periodic material, beam lattice, tetrachiral material.

1. Introduction

The transport of mechanical energy is a challenging corollary issue of the acoustic wave propagation in cellular materials with periodic microstructure. Indeed, the microstructure can determine significant anisotropies in the material behaviour. As immediate geometric consequence, the dispersion properties of microstructured materials seldom admit pure longitudinal and transversal waveforms, which are typical of isotropic media. More often, the geometric polarization of the acoustic waves is a non-trivial function of the microstructural parameters, with significant dependence on the propagation direction. Consequently, the wavefronts tend to be not-spherical and the direction of the energy fluxes and velocities does not necessarily coincide with the wavevector $\mathbf{k}$ of the propagating waves [1].

The balance laws governing the transport of mechanical energy are traditional fields of theoretical and applied research, dating back to the pioneer doctoral studies by Nikolay A. Umov in the late nineteenth century [2]. The mechanical energy transfered by acoustic waves, in particular, attracted the attention of eminent scientists all throughout the twentieth century. Among the others, Leon Brillouin studied the mechanical energy transfered between two adjacent cells of a periodic crystal lattice, and related its flux density to the particle velocities through the concept of characteristic impedance [3, 4]. Almost concurrently, Maurice A. Biot established some general theorems relating the group wave velocity and the energy velocity in anisotropic, non-homogeneous, non-dissipative media [5]. Over the last decades, specific issues related to the transport of mechanical energy have been treated in different monographs concerning anisotropic elastic solids [6], viscoelastic heterogeneous solids and fluids [7], anisotropic, anelastic, porous and electromagnetic materials [8], viscoelastic layered media [9]. Occasional but sharp attention has been specifically devoted to the flow of mechanical energy in periodic systems, including crystal lattices [10, 11] and structural assemblies [12, 13].

The essential idea, shared by the largest majority of literature studies, is that strict formal and substantial analogies can be established between the radiation of electromagnetic energy and the transfer of mechanical energy [14]. According to this standpoint, the *Poynting vector* – originally defined in the Maxwell theory of electromagnetism – can systematically be replaced by the *Umov-Poynting vector* $\mathbf{s}$ in solid mechanics [2, 15]. In a continuous material, the Umov-Poynting vector is a local quantity, expressing the mechanical power density related to the velocity field of a certain natural motion. In the specific case of natural wave motions, the Umov-Poynting vector depends on the dispersion relation $\omega(\mathbf{k})$ and is a quadratic function of the waveform $\boldsymbol{\psi}(\mathbf{k})$. Most significantly, the $\mathbf{s}$-vector accounts also

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