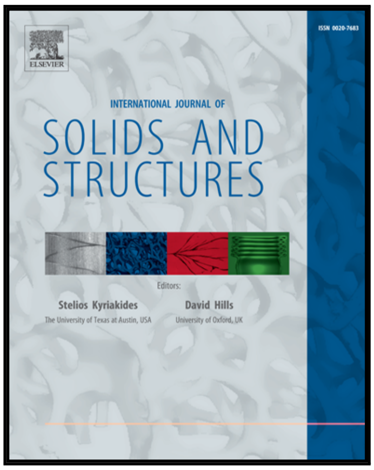
Accepted Manuscript



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PII: S0020-7683(18)30128-8
DOI: 10.1016/j.ijsolstr.2018.03.020
Reference: SAS 9945

To appear in: *International Journal of Solids and Structures*

Received date: 15 December 2017
Revised date: 13 March 2018
Accepted date: 19 March 2018

Please cite this article as: V.P. Tran, S. Brisard, J. Guilleminot, K. Sab, Mori–Tanaka estimates of the effective elastic properties of stress-gradient composites, *International Journal of Solids and Structures* (2018), doi: 10.1016/j.ijsolstr.2018.03.020

Mori–Tanaka estimates of the effective elastic properties of stress-gradient composites

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Abstract

A stress-gradient material model was recently proposed by Forest and Sab [*Mech. Res. Comm.* **40**, 16–25, 2012] as an alternative to the well-known strain-gradient model introduced in the mid 60s. We propose a theoretical framework for the homogenization of stress-gradient materials. We derive suitable boundary conditions ensuring that Hill–Mandel's lemma holds. As a first result, we show that stress-gradient materials exhibit a softening size-effect (to be defined more precisely in this paper), while strain-gradient materials exhibit a stiffening size-effect. This demonstrates that the stress-gradient and strain-gradient models are not equivalent as intuition would have it, but rather complementary. Using the solution to Eshelby's spherical inhomogeneity problem that we derive in this paper, we propose Mori–Tanaka estimates of the effective properties of stress-gradient composites with spherical inclusions, thus opening the way to more advanced multi-scale analyses of stress-gradient materials.

Keywords: Boundary Conditions, Elasticity, Homogenization, Inhomogeneity, Micromechanics, Stress-gradient

1. Introduction

Due to its lack of material internal length, classical elasticity fails to account for size effects frequently exhibited by e.g. nanomaterials. Generalized continua, which were introduced throughout the 20th century have the ability to overcome this shortcoming. The literature on generalized continua is very rich, and we only point at the most salient features of some models, in order to contrast them with the newly introduced stress-gradient model (Forest and Sab, 2012; Sab et al., 2016). The interested reader should refer to e.g. Askes and Aifantis (2011) for a more thorough overview. Higher-order and higher-grade models (to be discussed below) on the one hand share the same underlying idea: their strain energy mixes two or more strain variables which are not dimensionally homogeneous, effectively introducing material parameters that must be homogeneous to their ratio. Non-local models, on the other hand, assume that the local stress at a material point is related to the strains in a neighborhood of this material point (Eringen, 2002); clearly, the *size* of this neighborhood then defines a material internal length.

The Cosserat model (Cosserat and Cosserat, 1909) is probably the earliest example of generalized continua. It belongs to the class of higher-order continua, where additional degrees of freedom (namely, rotations) that account for some underlying microstructure are introduced at each material point. Elastic-plastic extensions of this model have been successfully used to explain the formation of finite-width shear bands in granular media (Mühlhaus and Vardoulakis, 1987). Other examples of higher-order continua are the so-called micromorphic, microstretch and micropolar materials (Eringen, 1999).

The strain-gradient model was introduced by Mindlin (1964) [and recently revisited by Broese et al. (2016)] as the long wavelength approximation of a more general material model for which a micro-volume is attached to any material point (Mindlin, 1964); it is the most simple example of higher-grade continua, in which the elastic strain energy depends on the strain and its first gradient. Mindlin discussed three equivalent forms of this theory (Mindlin and Eshel, 1968); he later introduced second gradient models in order to account for cohesive forces and surface tensions (Mindlin, 1965). The general first-gradient model requires in the case of isotropic, linear elasticity five additional material constants besides the two classical Lamé coefficients (Mindlin and Eshel, 1968) [this was recently questioned by Zhou et al. (2016), who introduced a subclass of isotropic materials for which only three additional material constants are needed]. Identification of strain-gradient material models can therefore be a daunting task, and Altan and Aifantis (1992, 1997) introduced a simplified model requiring only one material internal length. This model was later refined by Gao and Park (2007), who clarified the associated boundary conditions.

Having in mind the work of Mindlin and others on strain-gradient materials, it is natural to follow the path towards stress-gradient materials. While the formulation of strain-gradient models relies on the elastic strain energy depending on the strain and its first-gradient, stress-gradient models rely on the complementary elastic strain energy depending on the stress and its first gradient. From this perspective, the Bresse–Timoshenko

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