ARTICLE IN PRESS
JID: SAS [m5G;March 31, 2018;10:20]
International Journal of Solids and Structures 000 (2018) 1–19
Contents lists available at ScienceDirect
International Journal of Solids and Structures
journal homepage: www.elsevier.com/locate/ijsolstr

Models of the shells having ribs, reinforcement plates and cutouts

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ARTICLE INFO

Article history:
Received 12 August 2017
Revised 13 January 2018
Available online xxx

Keywords:
Shells of a stepped variable thickness
Mathematical model
Supported shells
Shells with cutouts
Boundary conditions
Structural anisotropy method

ABSTRACT

The work considers shells of a stepped variable thickness when a thickness variation is set by means of unit bar graph functions equal to a difference of two unit functions. It enables for considering ribs, reinforcement plates and cutouts in one structure; a rib and shell contact is arranged along a strip.

It is shown on the basis of the variational procedure for derivation of equilibrium equations that the boundary conditions (free boundary) are fulfilled automatically at the lateral surface of ribs and boundary of the cutouts when solving boundary-value problems.

1. Introduction

Thin-walled shell structures are widely used in various technical fields as well as in construction. To stiffen them more fully they are supported by ribs, however, for technological reasons they can also have cutouts. Thus, a shell model containing ribs, cutouts and reinforcement plates taking into account the main factors influencing the stress and strain state of such a structure is necessary.

The main ideas of calculation of ribbed shells were stated in the late 1940′s by Lurie (1948) and Vlasov (1949) who founded the two main approaches to the discrete introduction of ribs. V. Z. Vlasov considered a shell and a system of ribs supporting it as a contact problem. A. I. Lurie considered both a shell and a system of ribs supporting it as a whole. He derived equilibrium equations from the functional minimum condition with regard to the total potential deformation energy. Both A. I. Lurie and V. Z. Vlasov considered that the ribs representing Kirchhoff–Clebsch rods interact with a shell along the line. At the same time locations of ribs were set by means of delta-functions. In the case of such an approach it is assumed (Karmishin et al., 1975) that:

- an impact of the shell-supporting ribs on a shear and torsion of a shell mid-surface is neglected;
- deformation of supports is described by ratios of the linear stress state without considering their interference.

The third approach to the introduction of ribs is based on rib stiffness smearing over the shell surface.

The first two of the above approaches to the introduction of ribs are generally applied in the majority of the works of the authors, thus taking into account the discreteness of their location (Amiro and Zarutsky, 1980; Greben, 1965; Mikhailov, 1980; Mileikovskii and Gretchaninov, 1969; Terebushko, 1964; Timashev, 1974), and consideration of a ribbed shell as a contact system was considered in works (Bushnell, 1971; Maiborodina and Meish, 2013; Qatu et al., 2012). The same two approaches are applied to support openings in a shell (Maximyuk et al., 2014; Guz et al., 2009). The multiply-connected domain is considered in the presence of cutouts in a shell and the problem is solved by the finite elements method (Guz et al., 2009).

The work of Van der Neut (1947) was one of the first works in the field of stability of eccentrically supported closed cylindrical shells in which the author denoted the importance of eccentricity of stiffening ribs in case of loss of stability under axial compression.

The works (Jaunky et al., 1996; Kidane et al., 2003) single out three groups of approaches to consideration of stiffening ribs in the structure. These are the discrete approach (Wang and Hsu, 1985), the branched plate/shell approach, as well as the smeared stiffness approach (Dow et al., 1953; Troitsky, 1976; Reddy et al., 1985; Jaunky, 1992 and others).

The familiarity with the works where the ribs are introduced by means of delta-functions gave rise to distrust in the author due to the following reasons:

- Delta-functions, being limiting functions, do not have a graphic representation, therefore a ribbed shell cannot be represented graphically in case of such an approach to the introduction of ribs. Consequently, such an approach to the

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https://doi.org/10.1016/j.ijsolstr.2018.03.024

introduction of ribs is necessary so that it makes it possible to derive ratios which are analytically taken on intuitively by a passage to the limit for shells supported by narrow ribs when the places of connection of the ribs to the shell are set by means of delta-functions.

– When introducing ribs by means of delta-functions, the ribs, when crossing, do not have interference, that is they are not fixed rigidly to each other, and, therefore, they considerably reduce stiffness of a supporting ribbing.
– The experimental investigations of Klimanov and Timashev (1985) have shown a significant impact of a cross sectional ribbing on a shear and, particularly, on torsion of a shell mid-surface.

The familiarity with the work of Vainberg and Roytfarb (1965) where a panel with different parts having different thickness was considered, and this difference of thicknesses is set by means of a unit function, suggested that a ribbed shell can be set as a shell of a stepped variable thickness with introduction of unit bar graph functions $\bar{\delta}(x-x_j)$, $\bar{\delta}(y-y_i)$ as differences of two unit functions. In this case rib and shell contact is arranged along a strip with a stiff connection of ribs at their crossing, and shear and torsional stiffness of ribs are taken into account (Ilyin and Karpov, 1986; Karpov and Petrov, 1975; Karpov et al., 2002; Karpov, 2010).

In the late 1960′s, Zhilin (1971) noticed that in the case of the approach to ribbed shells of A. I. Lurie, the shell theory and the rod theory, hypotheses of which are not quite compatible, are used in equilibrium equations. Therefore, he suggested describing the operation of a shell and ribs by ratios of the shell theory.

A ribbed shell as a shell of a discrete and variable thickness was also considered in the works of Endzhievsky (1982) and Preobrazhensky (1981), however, an analysis of the approach to ribbed shells as to the shells of a stepped variable thickness was not performed in the above listed works.

The analytical analysis of such an approach performed by the author has shown the following:

– as a special case, when a rib width tends to be 0, ratios are obtained in the limit when ribs are set by means of delta-functions. At the same time, it does not follow from the variational equation that shear and torsional stiffness of ribs are neglected, and the ribs when crossing each other do not interact (Karpov, 1999).
– The approaches to ribbed shells of A. I. Lurie and V. Z. Vlasov are equivalent (Karpov, 1999).
– Boundary conditions of the free boundary are fulfilled automatically at the lateral surface of ribs when solving boundary-value problems (Karpov et al., 2002).
– If a height of ribs is taken as negative, the shells with cutouts are obtained. The free boundary condition is fulfilled automatically along the cutout boundary for the shells with through cutouts, and the shell stiffness is taken as zero in places of cutouts. Thus, a problem for a simply connected domain is obtained for the shells with cutouts (Karpov et al., 2002; Karpov, 2010).
– When considering the shells of a stepped variable thickness, the boundaries of cutouts can be supported by stiffening ribs without any difficulties (Karpov et al., 2002).
– The conducted computational experiment for the investigation of stability of ribbed shells when the ribs are introduced along the line by means of delta-functions and along the strip by means of unit bar graph functions has shown that failure to take account of shear and torsional stiffness of ribs, their interference when crossing leads to significant errors. Additionally, when the ribs are introduced along the line, and all rib stiffness is concentrated along the line, the durability investigation in this case also leads to significant errors (Karpov et al., 2002).
– Taking into account boundary conditions at the lateral surface of ribs (boundary of the cutouts), the most exact scheme of the structural anisotropy method is obtained taking into account shear and torsional stiffness of ribs. This scheme is applied for the shells which are supported by close-spaced ribs (have a large number of cutouts) when observing some criteria (Karpov et al., 2002; Karpov, 2010).
– Ribs while setting them by means of unit bar graph functions can be directed in parallel to coordinate lines or at an angle to them, and can have variable height, and ribs, and cutouts can have a nonlinear form (Karpov, 2010).
– Consideration of transverse shears is essential for ribbed shells (Karpov et al., 2002).

Thus, consideration of a ribbed shell as a shell of a stepped variable thickness enables for taking into account various factors influencing the stress and strain state and stability of a shell to the fullest extent possible, and, as a special case, to obtain ratios when introducing ribs by means of delta-functions. Additionally, in this case a shell not only can be supported by stiffening ribs and reinforcement plates, but also loosened by cutouts boundaries of which can be supported by ribs.

The purpose of the given work is to develop a nonlinear mathematical model of deformation of shells of a stepped variable thickness.

2. Methodology used and related equations

2.1. Geometrical and physical ratios of thin shells

The classical shell theory of positive Gaussian curvature is considered taking into account transverse shears. The mid-surface of a shell of h thickness is taken as coordinate surface. Axes$x$, y of the orthogonal coordinate system are directed along lines of the shell principal curvatures. Axis z is orthogonal to the mid-surface and directed to concavity.

In this case, geometrical ratios (strain-displacement relations) in the mid-surface take the following form (Grigolyuk and Kabanov, 1978; Novozhilov, 1962):

$$\begin{aligned}
\varepsilon_x &= \frac{1}{A}\frac{\partial U}{\partial x} + \frac{1}{AB}\frac{\partial A}{\partial y}V - k_x W + \frac{1}{2}\theta_1^2,\\
\varepsilon_y &= \frac{1}{B}\frac{\partial V}{\partial y} + \frac{1}{AB}\frac{\partial B}{\partial x}U - k_y W + \frac{1}{2}\theta_2^2,\\
\gamma_{xy} &= \frac{1}{A}\frac{\partial V}{\partial x} + \frac{1}{B}\frac{\partial U}{\partial y} - \frac{1}{AB}\frac{\partial A}{\partial y}U - \frac{1}{AB}\frac{\partial B}{\partial x}V + \theta_1\theta_2,
\end{aligned} \tag{1}$$

where A, B — Lame parameters;

$$\theta_1 = -\left(\frac{1}{A}\frac{\partial W}{\partial x} + k_x U\right), \quad \theta_2 = -\left(\frac{1}{B}\frac{\partial W}{\partial y} + k_y V\right).$$

Besides, if transverse shears are taken into account (Timoshenko–Reissner model), we shall have:

$$\gamma_{xz} = k\, f(z)\, (\Psi_x - \theta_1), \quad \gamma_{yz} = k\, f(z)\, (\Psi_y - \theta_2).$$

Here $f(z)$ — function characterizing τ_{xz}, τ_{yz} stress distribution over the shell thickness.

Deformations at points positioned at z distance from the coordinate surface are expressed by the following ratios:

$$\varepsilon_x^z = \varepsilon_x + z\chi_1, \quad \varepsilon_y^z = \varepsilon_y + z\chi_2, \quad \gamma_{xy}^z = \gamma_{xy} + 2z\chi_{12},$$

Please cite this article as: V.V. Karpov, Models of the shells having ribs, reinforcement plates and cutouts, International Journal of Solids and Structures (2018), https://doi.org/10.1016/j.ijsolstr.2018.03.024