International Journal of Solids and Structures 144–145 (2018) 137–144

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Eigenstrain toughening in presence of elastic heterogeneity with application to bone

Z. Wang[a,b], D. Vashishth[b,c], R.C. Picu[a,*]

[a] Department of Mechanical, Aerospace and Nuclear Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States
[b] Center for Biotechnology and Interdisciplinary Studies, Rensselaer Polytechnic Institute, Troy, NY 12180, United States
[c] Department of Biomedical Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States

ARTICLE INFO

Article history:
Received 2 June 2017
Revised 13 March 2018
Available online 26 April 2018

Keywords:
Transformation toughening
R-curve, Elastic heterogeneity
Bone
Dilatational bands

ABSTRACT

Transformation toughening has been used in commercial products for several decades in order to increase the toughness of brittle materials. Composites made from an elastic matrix and elastic-plastic inclusions similarly exhibit increased toughness and R-curve behavior due to the residual stress induced in the wake of the crack tip by the unloaded, plastically deforming fillers. These two mechanisms, in which the eigenstrains in the wake of a major crack lead to toughening, belong to the same class. In this study, we investigate the effect of the elastic heterogeneity of the matrix on such toughening mechanisms and observe that increasing the elastic heterogeneity amplifies the effect. The analysis is relevant for bone, which is a highly heterogeneous hierarchical material, in which localized plastic deformation has been recently shown to occur at dilatational bands. Understanding toughening in bone is a subject of current interest in the context of age-related fragility. The heterogeneity-enhanced eigenstrain toughening effect is of interest for a broad range of engineering applications.

1. Introduction

Toughening of brittle materials has been studied for almost half a century and a number of engineering solutions have been developed and adopted industrially (Pearson et al., 2000; Qin and Ye, 2015). Brittle materials fail in tension by the propagation of a major, critical crack. Failure occurs in compression either through wedging followed by sample splitting produced by a major crack that grows in the direction of the principal compressive load (the wing crack mechanism) (Cannon et al., 1990) or, in presence of confinement, through the formation of a shear band which links through a large number of microcracks produced during preliminary loading (Jaeger et al., 2007).

Toughening mechanisms are sought to enhance the material toughness measured in tension, which is typically much lower than that in compression. The main toughening mechanisms are crack bridging, crack deflection, crack pinning and transformation toughening. Crack bridging refers to the formation of ligaments behind the crack tip which restrict the crack opening displacement and hence lead to a reduction of the effective crack tip stress intensity factor (Swanson et al., 1987; Erdogan and Joseph, 1989; Evans and Hutchinson, 1989). Crack deflection is related to the presence of material heterogeneity. Strong obstacles to crack propagation lead to crack deflection, which increases the tortuosity of the crack path and the roughness of crack surfaces (Faber and Evans, 1983; Ahn et al., 1998). Crack pinning is similarly caused by material heterogeneity. Considering the crack to be planar and the material toughness to be spatially non-uniform, crack growth is slower in regions of higher resistance and the crack front becomes rough (Spanoudakis and Young, 1984). Transformation toughening is a more complex mechanism associated with the presence of material sub-domains that undergo a phase transformation under the action of the stress field in the region of the crack tip. This transformation produces eigenstrains in the surrounding elastic and non-transforming matrix, which, in turn, act on the crack, reducing the effective stress intensity factor. While the transformation occurs mainly in front of the crack, the toughening effect is due to the eigenstrains of the transformed sub-domains located in the wake of the tip. Hence, an R-curve effect is observed, the effective toughness increasing as the crack grows. The toughness eventually reaches a plateau once the wake is fully formed (Sakai et al., 1988). The transformation toughening mechanism has been used to toughen ceramics by the incorporation of yttria-stabilized zirconia particles in the respective material (Garvie et al., 1975; Budiansky et al., 1983; Ortiz, 1987). These particles are metastable and undergo a phase transformation when loaded by the large stress field in the vicinity of a crack tip. The eigenstrain produced

* Corresponding author.
E-mail address: picuc@rpi.edu (R.C. Picu).

https://doi.org/10.1016/j.ijsolstr.2018.04.019

during this transformation leads to the toughening effect described above.

In composites with elastic matrix and elastic-plastic inclusions, localized plastic deformation in fillers is expected in the vicinity of the crack tip. This energy dissipation leads to an increase of the critical energy release rate. As the crack advances, the inclusions that deform plastically in the process zone of the crack move in the wake of the tip and are unloaded. Since the surrounding matrix is elastic, these inclusions are forced to return to a strain state close to the initial undeformed configuration and consequently, an eigenstress is produced in the matrix. This mechanism is qualitatively similar to that of transformation toughening or process zone toughening and has similar effects on the crack. The common ground of the two mechanisms is that toughening is associated with the occurrence of eigenstrains which act on the crack. Here we demonstrate the impact of such mechanisms on fracture toughness and further analyze the effect of rendering the matrix elastically heterogeneous. A finite element (FEM) model is developed aimed to isolate the contribution of eigenstrain toughening from the other toughening mechanisms expected in heterogeneous materials, specifically, crack deflection and crack pinning. We also inquire whether rendering the matrix material heterogeneous may enhance the effectiveness of the eigenstrain toughening mechanisms.

This investigation is motivated in part by the need to understand toughness of bone. Many of the toughening mechanisms mentioned above have been discussed in relation to bone toughness (Vashishth et al., 1997; Nalla et al., 2003, 2004; Tang and Vashishth, 2007; Koester et al., 2008; Launey et al., 2010). Crack deflection has been observed on the microscale, especially for cracks transverse to osteons. Crack deflection leads to rough crack surfaces, interlocking of these asperities and enhanced resistance to crack growth in both mode I and mode II (Koester et al., 2008). Recent studies (Pro et al., 2015; Abid et al., 2018) on stochastic microstructures of bio-composite show that crack deflection toughening depends on the statistical variability of the microstructure. Specifically, in the case of nacre, the toughness decreases as the microstructural variability increases. Nucleation of microcracks in front of a major crack tip followed by coalescence with the main crack was also documented (Vashishth et al., 1997; Tang and Vashishth, 2007). Uncracked ligaments and collagen fibrils bridging have been observed on micrometer to submicrometer length scales (Nalla et al., 2003, 2004; Koester et al., 2008). Poundarik et al. (2015) demonstrated that microcracking controls and interacts with other toughening mechanisms in bone. However, eigenstrain toughening has not been considered so far as a potential toughening mechanism in bone.

Recently, the occurrence of submicron diffuse damage regions was reported in fatigued cortical bone samples (Diab and Vashishth 2005; Diab et al., 2006; Vashishth, 2007). Sub-domains in which inelastic deformation takes place are known as dilatational bands (Poundarik et al., 2012; Seref-Ferlengez and Basta-Pljakic, 2014). The specific mechanical behavior of bone within a dilatational band is largely unknown. However, due to co-localization with non-collagenous protein (NCPs) complexes (i.e. osteocalcin, osteonectin and osteopontin), it has been suggested that the dilatational bands formation is a result of protein complex denaturation (Poundarik et al., 2012). Also, nanoindentation experiments performed in such regions indicate an effective modulus 13% lower than that of the surrounding material (Poundarik et al., 2012). This level of reduction is probably an underestimate of the actual modulus reduction since probing was performed in dilatational bands embedded in undamaged bone and the effect of this confinement on the measured effective stiffness was not evaluated.

The physical picture proposed here is that the dilatational bands occurring in the vicinity of a major crack tip behave similar to elastic-plastic particles embedded in an elastic matrix of a composite. Before the dilatational bands form, the material is linear elastic. Subsequently, the material becomes plastic in the dilatational band sub-domains. Upon unloading, these sub-domains produce eigenstress, which in turn act on the crack leading to a reduction of the effective stress intensity factor and hence to toughening. This process takes place only upon unloading and hence is effective in the region behind the crack tip. In the following sections, we evaluate the magnitude of the toughening effect for several volume fractions of dilatational bands. Further, we investigate the effect of the elastic heterogeneity in regions of the model surrounding the dilatational bands (matrix elastic heterogeneity) on the toughening mechanism proposed. This investigation is motivated by the observation that bone is indeed a heterogeneous material (Nicolella et al., 2005; Tai et al., 2007; Thurner, 2009). The coefficient of variation of bone elastic moduli within a given sample and in absence of dilation bands is reported by Tai et al. (Tai et al., 2007) to be approximately 0.4, while a broader range, from 0.2 to 0.5, was inferred by Thurner (2009) for human cortical bone, based on literature data. This concept is also relevant for toughening of engineering brittle materials, case in which it is of interest to inquire whether material stochasticity may enhance the effect of the eigenstrain toughening mechanisms.

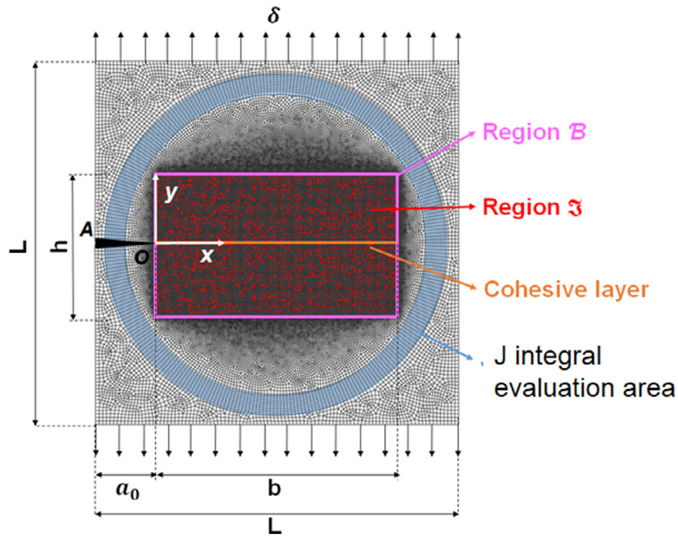

Fig. 1. Schematic representation of the compact tension specimen used in this study. $\mathcal{B}$ denotes the region where elastic-plastic/transformation sub-domains and/or matrix elastic heterogeneity are defined. The reunion of the transformation sub-domains in region $\mathcal{B}$ (red dots) is denoted by $\Im$. A crack of initial length a_0 is introduced along segment OA (O is at the origin of the coordinate system) and a cohesive zone is defined along plane $y = 0$ in front of the crack. The circular domain surrounding region $\mathcal{B}$ (shown in blue) is used for the calculation of the J integral. δ is the displacement boundary condition applied at the top and bottom boundaries. Heterogeneity is limited to region $\mathcal{B}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2. Model description

A schematic of the compact tension specimen used in this study is shown in Fig. 1. The model has dimensions L in the directions parallel and perpendicular to the crack. The initial crack length is $a_0 = L/6$ and the crack growth is restricted to the $y = 0$ plane containing the initial crack. Restricting the crack to be planar helps separate the effect eigenstrain toughening from that of other toughening mechanisms caused by heterogeneity. To enforce this restriction, cohesive elements are used along this plane, as indicated in Fig. 1. This set-up ensures that the crack remains planar and hence the crack deflection mechanism plays no role in the