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Calibration of tensile strength to model fracture toughness with distinct element method

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ABSTRACT

This study investigates the relation between two approaches for modeling fracture propagation. The first one is the classical approach, in which fracture propagates once the stress intensity factor exceeds a critical value, called fracture toughness. In the second approach, the fracture propagates once the tensile stress ahead of the fracture tip exceeds a critical value, called tensile strength. The purpose of this study is to examine the relation between the two approaches and to determine a methodology to make them equivalent. To address the goal, propagation of a radially symmetric fracture is first analyzed. A universal relation between the tensile strength and fracture toughness is obtained, which is then verified via a series of numerical examples. It is found that in order to capture the fracture toughness the tensile strength should be varied with respect to the mesh size and other material parameters. The developments are then applied to a three-dimensional distinct element code, which can be used in various applications involving modeling of a jointed and blocky material. An additional challenge with the distinct element code lies in the fact that the use of uniform value of tensile strength does not lead to a spatially uniform apparent fracture toughness. The latter is caused by mesh distortions and orientation of the elements relative to the fracture front. This problem is successfully addressed by introducing a variation of the tensile strength relative to local geometry of the mesh in the vicinity of the fracture front. The obtained result develop a procedure to accurately model fracture toughness in numerical methods that use tensile strength as a fracture propagation criterion.

1. Introduction

There are two primary approaches for modeling fracture propagation. The first approach is to use Linear Elastic Fracture Mechanics (LEFM) (Rice, 1968), in which fracture propagates when stress intensity factor at the fracture tip exceeds a critical value, called fracture toughness. The second approach involves using cohesive zone model (Barenblatt, 1962; Dugdale, 1960), in which there is a process zone ahead of the fracture tip that features a traction-separation law. The traction-separation law in the cohesive zone model is mainly determined by two parameters: tensile strength and fracture energy (area under the traction-separation law). The tensile strength represents the maximum tensile stress that the rock can sustain, while the fracture energy is related to the work needed to propagate the fracture. The LEFM approach can be seen as the far field limit for the cohesive zone model, i.e. both models can be made equivalent at distances much larger than the size of the cohesive zone by using a suitable relation between the fracture energy and fracture toughness.

In view of the two classical approaches described above, there is yet another method, which is commonly used to model fractures within the framework of distinct element method (Cundall, 1988; Hart et al., 1988). The rock is modeled as a collection of particles in the distinct element method. These particles are connected together by joints, which are characterized by tensile strength, normal and shear stiffness, friction, and cohesion. The tensile mode I fracture propagates if the tensile stress ahead of the fracture tip exceeds the tensile strength of the joint. A review of discrete element modeling techniques can be found in Lisjak and Grasselli (2014).

This study utilizes a particular distinct element method software called 3DEC, which is developed by Itasca Consulting Group Inc. (2013). This is a three-dimensional distinct element code that can be used in various applications involving modeling of rock as a jointed and blocky material, such as to model rock excavations,

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analyze slope stability, etc. Each block (or particle) is not rigid and can be modeled as a poroelastic material, but we focus exclusively on linear elastic materials. In this study we utilize 3DEC to examine its ability to model propagation of hydraulically-driven fractures. The latter are often induced deep in the subsurface to stimulate oil and gas production (Economides and Nolte, 2000). There is a variety of numerical algorithms that have been developed to model hydraulic fractures, see for instance summary of the classical approaches in Adachi et al. (2007). The distinct element modeling is a relatively new method, but it has already been used in multiple studies (Nagel et al., 2013; Damjanac and Cundall, 2016; Zhang and Dontsov, 2018).

The tensile strength criterion that is used in the distinct element method to propagate mode I fractures is different from the classical cohesive zone model. The primary difference arises from the fact that process zone is effectively restricted to a single element. Consequently, this leads to mesh dependence of the solution, which is reflected in the element size and shape dependence. So that two solutions that use the same value o f the tensile strength and that are calculated using different meshes are different. Recognizing that, 3DEC embeds the mesh size into the relation between the tensile strength and fracture toughness (Potyondy and Cundall, 2004)

$$\sigma_t = a\frac{K_{Ic}}{d^{1/2}}, \tag{1}$$

where σ_t is the tensile strength, K_{Ic} is mode I fracture toughness, d is the average element size, while $a = O(1)$ is a numeric parameter, which needs to be properly calibrated. The purpose of this study is to determine the value of a, so that the fracture modeled in 3DEC is equivalent to the fracture modeled using classical LEFM approach. Results of this study can also be applied to other methods, which utilize similar tensile stress-based propagation criterion.

In order to have a reference point, we first test predictions of 3DEC against reference solutions for a radial hydraulic fracture for different regimes of propagation, assuming $a = 1$, which is the default value for a. Comparison in the viscosity dominated regime allows us to verify the coupling between the fluid flow and elasticity. On the other hand, comparison of the solutions in the toughness dominated regime allows us to evaluate the propagation condition, which is the primary focus of this study. The initial comparison between 3DEC and the reference radial solution is summarized in Section 2. It is found that 3DEC is able to accurately model viscosity dominated radial fracture, while the toughness dominated fracture has a discrepancy in radius and the shape is not circular. To better understand the problem, Section 3 presents a 3DEC-like model for a hydraulic fracture propagating in the toughness dominated regime, which utilizes a propagation condition that is similar to 3DEC. This model is significantly faster than 3DEC and allowed us to calibrate the parameter a and to evaluate accuracy of the developed methodology for a wide range of parameters. Equipped with the understanding from the toy model, Section 4 addresses the problem of calibration of the parameter a for 3DEC. In addition, this section presents an approach to resolve the issue of non-circular fracture shape. Finally, it evaluates accuracy of the correction by comparing the results to the reference solution for a radial fracture for various parameters.

2. Radial hydraulic fracture with 3DEC: initial results

As a starting point in evaluating 3DEC for fracture applications, we consider the simplest case of a radial hydraulic fracture. To simplify mathematical expressions, it is convenient to introduce scaled

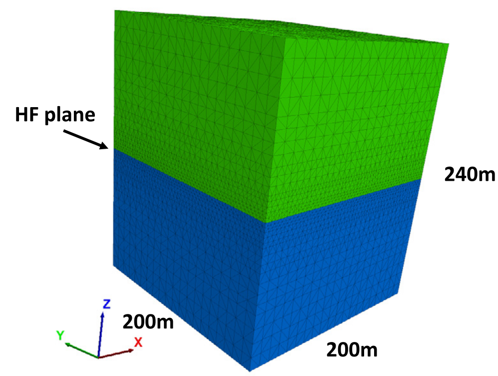

Fig. 1. 3DEC model setup.

material parameters as

$$\mu' = 12\mu, \qquad E' = \frac{E}{1-\nu^2}, \qquad K' = 4\left(\frac{2}{\pi}\right)^{1/2} K_{Ic}, \tag{2}$$

where μ is the fracturing fluid viscosity, E is the Young's modulus, ν is the Poisson's ratio, and K_{Ic} is the mode I fracture toughness of the rock. The solution also depends on the injection rate Q_0 and the injection time t. We consider three sets of material parameters, which correspond to the viscosity dominated regime, toughness dominated regime, and the transition region (Madyarova, 2003; Dontsov, 2016). For the viscosity dominated regime, the parameters are

$$E' = 20\times10^9 \text{ Pa}, \quad \mu' = 0.2 \text{ Pa·s}, \quad K' = 3\times10^6 \text{ Pa·m}^{1/2},$$
$$Q_0 = 0.01 \text{ m}^3/\text{s}, \quad t = 1000 \text{ s}. \tag{3}$$

For the transition region, we use

$$E' = 20\times10^9 \text{ Pa}, \quad \mu' = 0.2 \text{ Pa·s}, \quad K' = 3\times10^6 \text{ Pa·m}^{1/2},$$
$$Q_0 = 0.01 \text{ m}^3/\text{s}, \quad t = 1000 \text{ s}. \tag{4}$$

Finally, for the toughness regime, the parameters are selected as

$$E' = 5\times10^9 \text{ Pa}, \quad \mu' = 0.01 \text{ Pa·s}, \quad K' = 3\times10^6 \text{ Pa·m}^{1/2},$$
$$Q_0 = 0.01 \text{ m}^3/\text{s}, \quad t = 1000 \text{ s}. \tag{5}$$

As indicated in Madyarova (2003) and Dontsov (2016), in the absence of leak-off, as considered in this study, the regime of propagation is determined by the dimensionless time

$$\tau = \frac{t}{t_{mk}}, \qquad t_{mk} = \left(\frac{\mu'^5 E'^{13} Q_0^3}{K'^{18}}\right)^{1/2}.$$

If $\tau \lesssim 4.5\times10^{-2}$, then the fracture propagates in the viscosity dominated regime. If $\tau \gtrsim 2.6\times10^6$, then the fracture propagates in the toughness dominated regime. If the value of τ is in between these values, then the fracture is in the transition region. For the above three cases (3)–(5), we have $\tau = \{0.12, 217, 1.8\times10^6\}$. Given that the boundaries between the regimes are determined by the values of τ that vary on a logarithmic scale, the case with $\tau = 0.12$ can be practically considered as the viscosity dominated case, $\tau = 1.8\times10^6$ is practically the toughness dominated case, while $\tau = 217$ is approximately in the middle between the boundaries of the regimes (on a logarithmic scale).

Fig. 1 shows 3DEC model setup. The fracture plane is located between two elastic blocks connected together by a joint. Domain size in the fracture plane is 200×200 m. Initial compressive stress is taken as 20 MPa, while the pore pressure is chosen as 10 MPa. The reference radial solution does not depend explicitly on Poisson's ratio, while 3DEC needs it for calculations. The value of $\nu = 0.3$ is selected for computations. Tensile strength for each joint element is assigned according to (1) with $a = 1$.