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Numerical study on the dynamic properties of wrinkled membranes

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ABSTRACT

A membrane is flexible and tends to wrinkle under small compression. The wrinkling deformation adversely affects its surface profile and mechanical behavior and it should be controlled. Identifying the wrinkle-influencing factors and analyzing their effects on the performance of a membrane may help to devise a better way to reduce or eliminate the wrinkling deformation. In this paper, the authors discuss the wrinkle-influencing factors such as pre-stress, Poisson ratio, Young's modulus, thickness and boundary conditions, and numerically analyze their effects on the dynamic properties of a rectangular membrane under shear and a square membrane under corner loads with the membrane element previously proposed by the authors. Some interesting phenomena are noted in the analysis and they are discussed.

1. Introduction

A thin membrane is flexible with negligible bending stiffness and it tends to wrinkle under small compression. When wrinkled, a membrane almost loses its stiffness in the direction perpendicular to wrinkles and behaves differently from its original mechanical behavior before wrinkling. This property of membrane should be considered in the analysis of a membrane.

Existing models dealing with the above-mentioned wrinkling problem can be categorized into two types. One is based on the Tension Field theory assuming that a membrane has no bending stiffness and cannot resist any compression. Its derivatives include the modified deformation gradient (Roddeman et al., 1987; Miyazaki, 2006; Shaw and Roy, 2007), the modified strain tensor (Hornig and Schoop, 2003; Raible et al., 2005), the modified material constants (Ding and Yang, 2003), the modified constitutive relationship (Akita et al., 2007; Jarasjarungkiat et al., 2008, 2009; Yang et al., 2011; Zhao et al., 2011; Wang et al., 2014), the relaxed strain energy method (Haseganu and Steigmann, 1994; Atai and Steigmann, 1998, 2012, 2014; Epstein and Forcinito, 2001; Mosler and Cirak, 2009; Taylor and Steigmann, 2009; Taylor et al., 2014; Patil et al., 2015), the cable analogy (Stanuszek, 2003), etc.

The other category is based on the stability theory of plates and shells, assuming a thin membrane has a small bending stiffness and its wrinkles can be regarded as the buckling deformation under compression (Wang et al., 2008, 2010; Damil et al., 2013; Liu et al., 2013; Luo and Yang, 2014; Huang et al., 2015). This kind of wrinkling model can yield detailed out-of-plane deformation of a wrinkled membrane, and it usually employed the explicit time integration scheme for a more efficient convergence in the numerical analysis.

A wrinkled membrane can then be analyzed accurately with these models to study the morphological characteristics of the wrinkling deformation and the static or dynamic behavior of a membrane. Research on the wrinkling morphology involved the wrinkling patterns (Miyamura, 2000; Wong and Pellegrino, 2006a, b, c; Lecieux and Bouzidi, 2010; Lan et al., 2014), their evolution (Wang et al., 2012, 2013a, b) or transition between the near-threshold (NT) and the far-from-threshold (FFT) regimes (Davidovitch et al., 2011; Taylor et al., 2015), and their interactions (Senda et al., 2015).

Research on the static behavior of a wrinkled membrane has been discussed in detail (Wang et al., 2016) and more attention is devoted here to its dynamic performance. Researchers working in this area mainly studied the dynamic behavior of membranes with different boundary conditions (Young et al., 2005; Li et al., 2012; Alsahlani and Mukherjee, 2013), initial conditions (Shin et al., 2006; Liu et al., 2013a, 2016), geometrical parameters (Plaut, 2009; Das Gupta and Tamadapu, 2013; Liu et al., 2013a; Wang et al., 2015), physical properties (Plaut, 2009; Liu et al., 2013b, 2016), pre-stress (Liu et al., 2013b, 2016), etc.

The boundary conditions discussed in the literature involve shear displacement along an edge of a rectangular membrane (Li et al., 2012), corner loads for a square membrane (Young et al., 2005), location of the eccentric circular areal constraint for a circular membrane (Alsahlani and Mukherjee, 2013), enclosed air pressure for an inflated circular membrane (Chaudhuri and Das Gupta,

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2014; Wang et al., 2015), support stiffness for a lenticular inflatable parabolic reflector (Liu and Jin, 2015), etc.

The initial conditions refer to the initial displacement (Liu et al., 2013a), initial velocity (Shin et al., 2006; Liu et al., 2016) and initial acceleration (Shin et al., 2006). Liu et al. (2013a, 2016) studied the dynamic behavior of a four-edges-simply-supported rectangular membrane with different initial displacements in z-direction and different initial velocities of a pellet as an impact load. Shin et al. (2006) investigated and discussed the influences of the translating velocity and acceleration on the dynamic performance of an axially moving membrane supported by two pairs of rollers.

The geometrical parameters of a membrane that were studied in the literature include the curvature (Das Gupta and Tamadapu, 2013), aspect ratio (Plaut, 2009; Liu et al., 2013a; Wang et al., 2015), etc. Das Gupta and Tamadapu (2013) studied the contribution of curvature to the dynamics of an isotropic elastic membrane. Plaut (2009), Liu et al. (2013b) and Wang et al. (2015) investigated the influence of aspect ratio on the vibration of flat annular membranes, rectangular orthotropic membranes with viscous damping and inflated beams.

The physical properties have an effect on the dynamic behavior of a membrane and some researchers showed interest in the influence of the orthotropic ratio (Liu et al., 2013a, 2016), areal density (Liu et al., 2016) and Poisson ratio (Plaut, 2009). The orthotropic ratio is the ratio of the modulus E_1 to the modulus E_2 for an orthotropic material. Liu et al. (2013a, 2016) studied the effects of the orthotropic ratio and areal density on the nonlinear vibration response of a pre-tensioned rectangular orthotropic membrane. Plaut (2009) investigated the influence of the Poisson ratio on the vibration frequencies of a flat annular membrane.

In addition, pre-stress is also an important factor that greatly influences the out-of-plane stiffness and the dynamic properties of a membrane. Liu et al. (2013a, 2016) studied the effect of pre-stress on the vibration response of a pre-tensioned rectangular orthotropic membrane.

Although some useful researches have been conducted on the dynamic analysis of membranes under the influence of parameters mentioned above, only a few of them (Li et al., 2012; Wang et al., 2015) involved the wrinkling problem. These reports treated the wrinkles of a membrane as the local buckling deformation in the numerical analysis, and they studied the influences of the shear displacement, aspect ratio and enclosed air pressure on the first-order natural frequency of flat and inflated membranes.

There is a lack of research on how the dynamic behavior of a wrinkled membrane is affected by its pre-stress, material constants (the Poisson ratio and Young's modulus), thickness and boundary conditions. These parameters can be closely associated with the out-of-plane stiffness of a membrane and they may play an important role in its wrinkling deformation and dynamic performance. This paper aims to numerically study the effects of these parameters (pre-stress, Poisson ratio, Young's modulus, thickness and boundary conditions) on the natural frequencies and mode shapes of a membrane subjected to a certain wrinkling deformation with the wrinkling model previously proposed by the authors (Wang et al., 2014, 2016). Some interesting phenomena are noted in the study and they are worth reporting.

The layout of this paper is as follows: Section 2 briefly introduces the wrinkling strain to quantify the wrinkling deformation and its similarity to the plastic deformation; Section 3 briefly introduces the wrinkling model previously proposed by the authors; Section 4 presents the finite element implementation of the wrinkling model; Section 5 gives the verification of the wrinkling model on the dynamic problems; Section 6 discusses the wrinkle-influencing factors and explores their effects on the dynamic behavior of thin membranes; Section 7 summarizes the main conclusions.

2. Description of wrinkles

2.1. Basic equations of an isotropic membrane

If a point P on the surface of a membrane is defined by the position vector $\boldsymbol{X}(\xi^1, \xi^2)$ on the reference (initial) configuration Ω_0, or the position vector $\boldsymbol{x}(\xi^1, \xi^2)$ on the current configuration Ω (in which ξ^1 and ξ^2 are the contravariant curvilinear coordinate), the covariant base vectors $\boldsymbol{G}_\alpha$ on Ω_0 and $\boldsymbol{g}_\alpha$ on Ω ($\alpha = 1, 2$) can be formulated as:

$$\boldsymbol{G}_\alpha = \frac{\partial \boldsymbol{X}}{\partial \xi^\alpha}, \quad \boldsymbol{g}_\alpha = \frac{\partial \boldsymbol{x}}{\partial \xi^\alpha} \tag{1}$$

The contravariant base vectors $\boldsymbol{G}^\alpha$ on Ω_0 and $\boldsymbol{g}^\alpha$ on Ω have the following relationship with the covariant base vectors $\boldsymbol{G}_\alpha$ and $\boldsymbol{g}_\alpha$:

$$\boldsymbol{G}^\alpha = G^{\alpha\beta}\boldsymbol{G}_\beta, \quad \boldsymbol{g}^\alpha = g^{\alpha\beta}\boldsymbol{g}_\beta \tag{2}$$

where $\beta = 1, 2$, $G^{\alpha\beta} = \boldsymbol{G}^\alpha \cdot \boldsymbol{G}^\beta$, $g^{\alpha\beta} = \boldsymbol{g}^\alpha \cdot \boldsymbol{g}^\beta$, $[G^{\alpha\beta}] = [G_{\alpha\beta}]^{-1}$, $[g^{\alpha\beta}] = [g_{\alpha\beta}]^{-1}$, $G_{\alpha\beta} = \boldsymbol{G}_\alpha \cdot \boldsymbol{G}_\beta$ and $g_{\alpha\beta} = \boldsymbol{g}_\alpha \cdot \boldsymbol{g}_\beta$. The nominal deformation tensor $\boldsymbol{F}$ and Green's strain tensor $\boldsymbol{E}$ are defined as

$$\boldsymbol{F} = \boldsymbol{g}_\alpha \otimes \boldsymbol{G}^\alpha \tag{3}$$

$$\boldsymbol{E} = \frac{1}{2}\left(\boldsymbol{F}^{\mathrm{T}} \cdot \boldsymbol{F} - \boldsymbol{I}\right) = E_{\alpha\beta}\boldsymbol{G}^\alpha \otimes \boldsymbol{G}^\beta \tag{4}$$

where $\boldsymbol{I}$ is the second order unit tensor and $E_{\alpha\beta} = \frac{1}{2}(g_{\alpha\beta} - G_{\alpha\beta})$.

For an elastic and isotropic membrane, the nominal second Piola–Kirchhoff (PK2) stress tensor $\boldsymbol{S}$ is derived from the constitutive relationship:

$$\boldsymbol{S} = \boldsymbol{C} : \boldsymbol{E} \tag{5}$$

where $\boldsymbol{C}$ is the elastic tensor. The nominal Cauchy stress tensor $\boldsymbol{\sigma}$ can be obtained from Eq. (6) as

$$\boldsymbol{\sigma} = J^{-1}\boldsymbol{F} \cdot \boldsymbol{S} \cdot \boldsymbol{F}^{\mathrm{T}} = \sigma^{\alpha\beta}\boldsymbol{g}_\alpha \otimes \boldsymbol{g}_\beta \tag{6}$$

where $J = \det(\boldsymbol{F})$.

2.2. Wrinkling deformation

A membrane usually has a small bending stiffness and it would buckle under compression (as shown in Fig. 1). This kind of buckling deformation is called wrinkle. When a membrane wrinkles, only tensile stress exists in the wrinkling direction while the stiffness and stress almost vanish in its perpendicular direction along with a certain amount of in-plane contraction (Fig. 1). Such mechanical properties of a wrinkled membrane are completely different from its behavior before wrinkling.

2.3. The tension field theory

The Tension Field Theory (TFT) assumes that a membrane has no bending stiffness and resistance to compression. It is uniaxially tensioned in the wrinkling direction after it is wrinkled or free of stress in any direction after it becomes slack. The stress state of a wrinkled or slack membrane can then be represented with the assumption as (Miyazaki, 2006):

$$\boldsymbol{t}_0 \cdot \boldsymbol{S} \cdot \boldsymbol{t}_0 \ or\ [\boldsymbol{U}^1]^{\mathrm{T}}[\boldsymbol{S}] = \begin{cases} \hat{S}^{11} > 0 & if\ wrinkled \\ \hat{S}^{11} = 0 & if\ slack \end{cases} \tag{7}$$

$$\boldsymbol{w}_0 \cdot \boldsymbol{S} \cdot \boldsymbol{w}_0 \ or\ [\boldsymbol{U}^2]^{\mathrm{T}}[\boldsymbol{S}] = \hat{S}^{22} = 0 \tag{8}$$

$$\boldsymbol{t}_0 \cdot \boldsymbol{S} \cdot \boldsymbol{w}_0 \ or\ [\boldsymbol{U}^3]^{\mathrm{T}}[\boldsymbol{S}] = \hat{S}^{12} = 0 \tag{9}$$