ARTICLE IN PRESS
JID: SAS [m5G;March 28, 2018;14:54]
International Journal of Solids and Structures 000 (2018) 1–11
Contents lists available at ScienceDirect
International Journal of Solids and Structures
journal homepage: www.elsevier.com/locate/ijsolstr

Direct transformation of the volume integral in the boundary integral equation for treating three-dimensional steady-state anisotropic thermoelasticity involving volume heat source

Y.C. Shiah[a,*], Nguyen Anh Tuan[a], M.R. Hematiyan[b]

[a] Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan 701, Taiwan
[b] Department of Mechanical Engineering, Shiraz University, Shiraz 71936, Iran

ARTICLE INFO

Article history:
Received 9 September 2017
Revised 12 March 2018
Available online xxx

Keywords:
Three-dimensional anisotropic thermoelasticity
Direct volume integral transformation
Boundary element method (BEM)
Volume heat source

ABSTRACT

In the direct formulation of boundary integral equation (BIE), thermal effect is present as extra integral, destroying the advantage of boundary modeling feature. The most appealing approach is to analytically transform the domain integral onto boundary such that the boundary modeling feature can be restored. Recently, the leading author has presented a direct transformation for two-dimensional anisotropic thermoelasticity, not relying on any domain distortion. However, due to mathematical complexity, such direct transformation has not been achieved for three-dimensional generally anisotropic thermoelasticity. Despite the importance of this topic in the BEM, the direct transformation has remained unexplored so far. As the first successful work, this paper presents the complete process to make this direct transformation for treating three-dimensional anisotropic thermoelasticity with implementation in an existing code. Additionally, this work also takes into account the presence of constant volume heat sources.

1. Introduction

Nowadays, anisotropic materials have been widely applied in various engineering disciplines. Undergoing thermal treatments or environmental temperature change, the anisotropic structures will generally have thermoelastic stresses/strains developed inside, leading to a decline of structural integrity. Thus, thermoelastic analysis of anisotropic materials has always remained to be an important topic in engineering practice.

As an efficient numerical tool, the boundary element method (BEM) is well known for its distinctive feature that only the boundary needs to be modeled. This is especially useful when it is applied to model three-dimensional problems with complicated geometries, generally requiring great modeling efforts for using domain solution techniques. However, for treating thermal effects, an additional volume/domain integral will appear in the BIE. Any attempt to directly integrate the extra volume integral will generally require domain discretization, destroying the distinctive feature of boundary modeling. To avoid the direct integration, there are several schemes proposed over the years, including the dual reciprocity method (DRM) the domain fanning approach (e.g. Camp and Gipson, 1992), the particular integral approach (e.g. Deb and Banerjee, 1990), the dual reciprocity method (e.g. Nardini and Brebbia, 1982), the exact transformation method (e.g. Rizzo and Shippy, 1977), the radial integration method (e.g. Gao, 2003), and the Cartesian transformation method (e.g. Hematiyan, 2007, Mohammadi et al., 2010). Another interesting new approach was proposed by Wen et al. (1998) to transform domain integral to the boundary. Sharing the similar manner as in the DRM, this technique uses radial base functions to approximate the body force term. As proposed by Sladek and Sladek (1984), another approach to avoid the domain integration is to consider the uncoupled thermoelasticity as special classes of general coupled thermoelasticity with sets of fundamental solutions for particular classes of uncoupled problems. Sladek et al. (1990) also presented an iterative approach to avoid direct integration of the domain integral.

Obviously, the exact transformation method, abbreviated here as ETM, appears to be not only analytically elegant but also numerically efficient since no further approximation is involved except for the numerical integration itself. For treating 2D generally anisotropic thermoelasticity, Shiah and Tan (1999) present an analytically transformed boundary integral equation, considering the presence of constant volume heat source. However, this transformation replies on domain distortion, where the anisotropic heat conduction is governed by the standard Poisson's equation. Also, Shiah et al. (2016) applied the ETM to treat 2D transient

* Corresponding author.
E-mail address: ycshiah@mail.ncku.edu.tw (Y.C. Shiah).

https://doi.org/10.1016/j.ijsolstr.2018.03.019

thermal problems. To reduce the algorithm complexity of accounting for domain distortion, Pasternak (2012) also presented Somigliana-type truly boundary integral equations, where the volume-to-surface integral transformation did not rely on any domain distortion for treating 2D anisotropic thermoelasticity. In contrast to the previous work in Shiah and Tan (1999) derived using the fundamental solution presented by Lekhnitskii (1981), the transformation in Pasternak (2012) was derived using the Stroh formalism and the theory of analytic functions. Without the presence of internal heat sources, the obtained integral equations in Pasternak (2012) contain only curvilinear integrals over the boundary and crack faces. Being based on the developed BIEs, the dual boundary element method was further employed in Pasternak (2012) to analyze the fracture of anisotropic thermoelastic solids. By extending the work (Pasternak, 2012), Pasternak et al. (2013) continued to develop Somigliana-type boundary integral equations for 2D thermoelectroelasticity of anisotropic solids with cracks and thin inclusions.

As compared with 2D works, pertinent 3D works are extremely scarce indeed. For treating the more general case of thermomagnetoelectroelasticity, Pasternak et al. (2016) derived Somigliana-type truly boundary integral equations for 3D thermomagnetoelectroelasticity of anisotropic solids. As in their previous work for 2D (Pasternak et al., 2013), neither volume integral needs to be evaluated nor domain mapping is involved. All kernels for a point heat are obtained explicitly using the Radon transform technique. Also, Pasternak et al. (2017) applied the developed BIEs to study 3D fracture problems for thermomagnetoelectroelastic solids. As an attempt to exactly transform the volume integral in the thermoelastic BIE, Shiah and Li (2015) presented solutions of an elliptic partial differential equation; however, the computational efficiency is another issue arising when implemented in the BEM. Despite the success of applying the ETM to treat isotropic thermoelasticity for both 2D and 3D, its extension to 3D anisotropic thermoelasticity had not been so successful until Shiah (2016) presented an exact volume-to-surface integral transformation for 3D by the domain mapping technique as in Shiah and Tan (1999) when no heat source was involved. Following this work, Shiah and Chong (2016) also implemented this approach to perform interior thermoelastic analysis of three-dimensional generally anisotropic bodies. Perhaps, it is worth mentioning that the transformation in the previous works (Shiah and Li, 2015) was based on the fundamental solutions of 3D generally anisotropic elasticity but not thermoelastic fundamental solutions considering point heat source (Pasternak et al., 2017); thus, no specific considerations need to be paid for thermal boundary conditions even heat sources are prescribed anywhere inside.

For the domain mapping process taken in Shiah (2016), computations of the transformed boundary integrals defined in the mapped domain appear to be less direct and more complicated. Adopting the idea of removing the process of domain mapping (Pasternak, 2012), Shiah et al. (2014) proposed a new direct transformation to treat 2D anisotropic thermoelasticity without heat source, where no coordinate transformation was involved. To the authors' best knowledge, such direct and exact transformation for 3D generally anisotropic elasticity has remained unexplored despite its importance for the BEM development. This is majorly because of the mathematical complexity of the Green's function of 3D anisotropic elasticity. This paper aims to present the complete process to make the direct volume-to-surface integral transformation for 3D anisotropic thermoelasticity when a constant volume heat source is present inside domain. All presented formulations have been successfully implemented in an existing code, based on quadratic isoparametric elements. To illustrate the veracity of all formulations as well as the successful implementation, a few benchmark examples are investigated in the end.

2. BIE of thermoelasticity

As has been well established and documented in many textbooks, the constitutive law between stresses σ_{ij} and strains ε_{ij} with thermal effects is governed by the well-known Duhamel–Neumann relation, that is

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - \gamma_{ij}\Theta, \quad (i, j, k, l = 1, 2, 3) \tag{1}$$

where Θ is the temperature change, C_{ijkl} are the material stiffness coefficients, and γ_{ij} represent the thermal muduli, given by $\gamma_{ij} = C_{ijkj}\,\alpha_{kl}$ with α_{kl} being the coefficients of linear thermal expansion. Under the steady-state condition, the anisotropic thermal field with constant heat source is governed by

$$K_{jk}\Theta_{,jk} = C_0, \tag{2}$$

where K_{jk} denotes the heat conductivity coefficients and C_0 is the constant heat generation rate.

For a linear elastic body with thermal effects in the domain Ω, the displacement u_j and traction t_j on the boundary surface Γ are cross-related with each other by the well known BIE as follows (Sladek and Sladek, 1984):

$$c_{ij}(\xi)u_j(\xi) + \int_\Gamma T^*_{ij}(\xi, \mathbf{x})u_j(\mathbf{x})d\Gamma(\mathbf{x}) = \int_\Gamma U^*_{ij}(\xi, \mathbf{x})t_j(\mathbf{x})\, d\Gamma(\mathbf{x}) + \int_\Omega \Theta(\mathbf{x})\gamma_{jk}U^*_{ij,k}(\xi, \mathbf{x})\, d\Omega(\mathbf{x}), \tag{3}$$

where $c_{ij}(\boldsymbol{\xi})$ are the geometric coefficients of the source point $\boldsymbol{\xi}$, and $U^*_{ij}(\xi, \mathbf{x})$ and $T^*_{ij}(\xi, \mathbf{x})$ are the fundamental solutions of displacements and tractions, respectively. The very early work for deriving the three-dimensional Green's function of anisotropic elasticity was contributed by Lifshitz and Rosenzweig (1947) to express it as a contour integral around a unit circle on an oblique plane. For applying the BEM to treat 3D elastostatics, numerical computation has been an interesting subject in the BEM community over the past few decades. Details about how the present work computes the fundamental solutions will be addressed in the next section after treatment of the extra volume integral is elaborated. As the main issue of the present work, the last integral in Eq. (3), denoted here by V_j for brevity, is a volume integral that needs to be transformed to the boundary. As has been presented previously by Pasternak et al. (2013) for 2D, the extra volume integral can be directly transformed onto the boundary without any coordinate transformation; however, the transformation process to be made here employs the fundamental solutions of 3D generally anisotropic elastostatics that do not involve any point heat source.

Before the complete 3D transformation process is elaborated, previous derivation of the direct transformation for 2D is reviewed first. Consider the following identity (Pasternak et al., 2013):

$$\int_\Omega (f_i K_{jk}\Theta_{,jk} - \Theta K_{jk} f_{i,jk})\, d\Omega = \int_\Omega [(f_i K_{jk}\Theta_{,j})_{,k} - (\Theta K_{jk} f_{i,k})_{,j}]\, d\Omega, \tag{4}$$

where f_i is the component of an arbitrary function f. As a result of applying the Green's second identity to the right hand side of Eq. (4), one immediately obtains (Pasternak et al., 2013)

$$\int_\Omega (f_i K_{jk}\Theta_{,jk} - \Theta K_{jk} f_{i,jk})\, d\Omega = \int_\Gamma (f_i K_{jk}\Theta_{,j} n_k - \Theta K_{jk} f_{i,k} n_j)\, d\Gamma, \tag{5}$$

In the previous work (Pasternak, 2012), no heat source was assumed. However, in engineering practice, it is quite often to involve volume heat generation due to chemical reaction or electrical heating in domain. For this, the constant source term C_0 in Eq. (2) is taken into account. By substituting Eq. (2) into Eq. (5),

Please cite this article as: Y.C. Shiah et al., Direct transformation of the volume integral in the boundary integral equation for treating three-dimensional steady-state anisotropic thermoelasticity involving volume heat source, International Journal of Solids and Structures (2018), https://doi.org/10.1016/j.ijsolstr.2018.03.019