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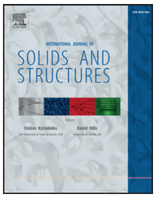

On the localisation of damage under pure bending using a nonlocal approach

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ABSTRACT

To reproduce the behaviour of quasibrittle materials mathematically, constitutive laws with softening are needed, which often leads to development of damage localisation bands. If a standard local formulation is used, this approach leads to a pathological mesh dependence, which can be eliminated by making use of alternative nonlocal formulations. The aim of this paper is to assess the localisation properties of damage models under pure bending using different nonlocal formulations; to permit a partially analytical treatment, the idealised case of pure bending is studied. Under these conditions, the localisation process starts at the tensile face of the beam, which belongs to the boundary of the domain on which the problem is solved. Consequently, localisation patterns are affected by the boundary treatment as well as other parameters, such as the characteristic length that defines the area contributing to the nonlocal averaging. This paper presents an analytical study of the onset of localisation of different nonlocal formulations for a beam under pure bending. In addition to it, the subsequent evolution of the localised solution is explored by numerical simulations, analysing the localisation bands spacing, the dissipated energy profile along the fracture plane and the Moment-$\frac{\phi}{L}$ diagrams, with $\frac{\phi}{L}$ being a parameter that represents the rotation that drives the loading process (ϕ stands for the relative rotation angle of the cross section and L for the beam length). An analysis of damage localisation on longer beams where damage localises in several areas is also carried out and, finally, the damage localisation due to shrinkage is studied as a more realistic example of the problem addressed here.

1. Introduction

Concrete is the most extended material in construction and, although studied for many decades, still gives rise to new developments and research. In particular, the time-dependent behaviour of concrete due to creep and shrinkage is an issue still under study, where many researchers are proposing methods and models to mathematically reproduce this process due to drying and aging (Bazant and Baweja, 2000; Gardner, 2004; Sakata and Shimomura, 2004; Havlásek and Jirásek, 2012; Havlásek, 2014; Bazant et al., 2015; Havlásek and Jirásek, 2016). Drying shrinkage on a concrete surface produces a uniformly distributed strain field on the element, which is a similar situation to a beam subjected to pure bending, where strain also varies in parallel planes. In this paper, the pure bending problem is used to study how different nonlocal formulations predict damage initiation and evolution, as an approach to the problem of shrinkage and creep.

In continuum damage mechanics, the use of constitutive laws with softening is widely extended to mathematically reproduce the behaviour observed with experiments. This type of models, when formulated locally, show a pathological dependence on the elements size. This issue can be addressed by simple adjustments that make some constitutive parameters dependent on the element size or by making use of more sophisticated enhancements that introduce an internal length, e.g. based on integral-type or gradient-type nonlocal formulations.

Unlike local schemes, nonlocal formulations evaluate the stress at each integration point using a state variable computed as a weighted average of the values at neighbour integration points, which eliminates the pathological mesh dependence of the local formulations. An overview of these methods can be found at Bažant and Jirásek (2002) and Planas et al. (1993).

Different alternatives can be used when defining a nonlocal formulation, e.g. using different weight functions to compute the contribution of each neighbour point to the nonlocal average. Nevertheless, one of the most interesting aspects of this problem is how to deal with nonlocal averaging near a body boundary which, as shown by Simone et al. (2004), can lead to a non-

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physical damage initiation away from the crack tip; a good study on this topic can be found in Havlásek et al. (2016). As shown by Jirásek et al. (2004) and analysed by Grassl et al. (2014), using standard averaging procedures may result in excessive spurious energy dissipation close to boundaries for notched specimens. To reduce these effects, alternative approaches have been proposed to obtain the average nonlocal value, some making the weight functions dependent on the distance to the boundary (Bolander Jr and Hikosaka, 1995; Krayani et al., 2009; Bažant et al., 2010) and some making them dependent on the stress state (Bažant, 1994; Jirásek and Bažant, 1994; Giry et al., 2011). Here, another interesting approach will be used, which was proposed by Polizzotto (2002) and Borino et al. (2003) and which will be referred to as the local complement method.

The aim of the present paper is to assess the localisation properties of nonlocal damage models under bending. To permit a partially analytical study and to display the fundamental properties of various formulations, the idealised case of uniform bending is studied. Therefore, the beam is considered to have a constant cross section and to be subjected to a uniformly distributed bending moment and zero normal force.

Six boundary treatments to compute the nonlocal values near the body boundaries are compared: standard scaling, local complement, two variations of a stress-based approach and two distance-based approaches. Furthermore, three weight functions are utilised to obtain the nonlocal values and their performance compared.

To analyse the possible combinations of boundary treatments and weight functions, two approaches are used. First, a simple analytical model is developed to observe how each of them computes the nonlocal strain profile of the beam cross-section under elastic conditions. This helps to understand why damage is developed earlier in some cases and helps to decide which alternative is more accurate compared to the expected elastic strain profile.

In addition to the analytical study, a finite element model is built to study the problem. When a nonlocal approach is used for modelling fracture on a beam subjected to pure bending, fracture numerically concentrates in equispaced bands, here the influence of each nonlocal formulation and the effect of the characteristic length R are investigated. Therefore, this model represents an infinite beam under pure bending; to do so, periodic boundary conditions are defined. This model is computed for several averaging schemes and several values of the characteristic length R to compare how each of them affects the localisation pattern. The localisation bands spacing is analysed, together with the Moment-$\frac{\phi}{L}$ diagram, with $\frac{\phi}{L}$ being the parameter representing the rotation that drives the loading process in the model (ϕ represents the rotation angle of the cross section and L the beam length), and the dissipated energy profile along the cross-section where damage is developed. Additionally, the evolution of damage on longer beams, where localisation takes place in several areas at the same time, is addressed.

Finally, to study a more realistic problem, an example of the effect of shrinkage on the localisation of damage is presented, which is a common situation in most concrete structures and produces similar effects as pure bending on the problem. This is carried out using a staggered scheme, that is to say, computing two physical models in parallel; the first one reproduces shrinkage and feeds the second model, which reproduces damage evolution as a consequence of strains due to shrinkage.

2. Nonlocal formulations

When using local formulations of damage models, there exists a mesh-dependence that makes it difficult to ensure that the result obtained is fully representative of the problem under consideration. To avoid this problem, nonlocal formulations allow to evaluate damage using nonlocal values of strain, that is to say, using a value of the equivalent strain computed as an average value of the Gauss point under consideration and those neighbours at a distance that is defined by a parameter with the dimension of length, usually referred to as the nonlocal characteristic length, which will be here denoted as R, following the notation by Grassl et al. (2014). To obtain this average value, different formulations can be applied and, depending on them, the result can have significant consequences on the final solution of the problem. In this section, these differences are analysed for the onset of localisation on a beam under pure bending.

On one hand, to compute the average strain at a certain Gauss point, the contribution of each neighbour is weighted depending on its distance s to the Gauss point of interest. To obtain the weight α_0 for each contributing point, different weight functions can be used; in this study the following have been considered:

- Gaussian function
$$\alpha_0(s) = \exp\left(-\frac{s^2}{R^2}\right)$$
- Exponential function
$$\alpha_0(s) = \exp\left(-\frac{s}{R}\right)$$
- Bell-shaped function:
$$\alpha_0(s) = \left\langle 1 - \frac{s^2}{R^2}\right\rangle^2$$
where $\langle\cdot\rangle$ denotes the Macaulay brackets:
$$\langle x\rangle = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

The shape of these functions can be observed in Fig. 1. It is interesting to note that the bell-shaped function is equal to 0 for values of s greater than R, while the other two, Gaussian and exponential functions, are extended beyond R, with the first of which being extended up to $\approx 2.5R$ and the second up to $\approx 6R$.

On the other hand, when close to a boundary, the area considered for averaging the strain value extends out of the body limits; to face this issue, the weight function can be scaled using different techniques. In this study, the following scaling techniques are considered:

- Standard scaling
- Local complement
- Distance-based scaling
- Stress-based scaling

2.1. Description of the nonlocal scaling options

The nonlocal strain is obtained using the following expression:

$$\bar{\varepsilon}(\boldsymbol{x}) = \int_V \alpha(\boldsymbol{x}, \boldsymbol{\xi})\tilde{\varepsilon}(\boldsymbol{\xi})\, \mathrm{d}\boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{x}$ stands for the coordinates of the point where the nonlocal value is computed (it will be referred to as point of interest in the future), $\boldsymbol{\xi}$ for the coordinates of each of the points that contribute to the nonlocal value (which in the case of the bell-shaped function correspond to those at a distance $\leq R$; in the case of the Gauss function and the exponential function, these correspond to those at a distance $\leq 2.5R$ and $\leq 6R$, respectively) and $\tilde{\varepsilon}(\boldsymbol{\xi})$ for the equivalent strain at $\boldsymbol{\xi}$.

Therefore, this expression provides a weighted average of the local equivalent strain by integrating the value of the local equivalent strain $\tilde{\varepsilon}$ at each point $\boldsymbol{\xi}$ and applying a weight function α.

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