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A cell-based smoothed finite element method for multi-body contact analysis using linear complementarity formulation

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ABSTRACT

In this paper, the cell-based smoothed finite element using quadrilateral elements (CS-FEM) is used for 2D contact problems which are converted into linear complementarity problems (LCPs), which can be solved efficiently using the Lemke method. The modified Coulomb friction contact model with tangential strength and normal adhesion is considered, which models sticking-slipping, contacting-departing, and bonding-debonding processes, in a unified formulation. Smoothed Galerkin weak-form with contact boundary is deduced, in which the stiffness is implemented using the CS-FEM with 1 smoothing domain (1SD), 2SD, 3SD, 4SD, 8SD, and 16SD for each element. Contact interface equations are discretized through contact point-pairs that are constructed using a master-slave surface algorithm. Intensive numerical examples are given to investigate the effects of contact parameters on contact behaviors and examine the effectiveness of the proposed approach. The numerical results of CS-FEM models are compared with that of FEM-Q4 model, which demonstrates that all CS-FEM models are softer than FEM-Q4 model. The strain energy solutions, obtained using several CS-FEM models, are monotonically decreasing with the number of the SDs for each element increasing. The upper bound solutions in strain energy can be obtained using a CS-FEM-1SD model in our examples, while the lower bound solutions are obtained using CS-FEM-16SD model or FEM-Q4, with FEM-Q4 solution being the lowest.

1. Introduction

Modeling, simulation and analysis of contact problems are of great importance in many industrial applications (Wriggers, 2006; Johnson, 1985; Goryacheva, 1998), including knee joints, hip joints or tooth implants in the medical applications, impacts of cars against building structures, connections of structural members by bolts or screws, gear and bearing systems, and feet, tyres or wheels interacting with surfaces of roads or rails. The list goes on and on.

Contact problems are strongly nonlinear in nature. The strong nonlinearity is caused by non-smooth contact interfaces and uncertain contact region, and special techniques are needed for efficient treatment of such nonlinearity. Contact problems with complex boundaries are even more difficult to solve, and hence use of a discrete numerical method is often a must. Many techniques of treating the constraints of contact interface have been developed in the past to solve various types of contact problems, mainly including iterative technique and optimization technique. The former technique has been widely applied in many methods, such as Lagrangian multiplier (Bertsekas, 1982; Papadopoulos and Solberg, 1998; Hild and Renard, 2010; Baillet and Sassi, 2002; Carpenter et al., 1991), penalty method (Weyler et al., 2012; Biotteau and Jean-Philippe, 2012; Okabe and Kikuchi, 2010), the augmented Lagrange method combining the penalty and the Lagrange multiplier techniques (Heegaard and Curnier, 1993; Mijar and Arora, 2004). Recently, Areias et al., (2014) proposed a general algorithm for 3D which is shown to have quadratic convergence in the Newton–Raphson iteration for quasi-static contact problems with friction and cohesive laws. Areias et al., (2015) proposed a contact algorithm with explicit projection in the cone for finite displacement quasi-static problems, which possesses apparent advantage for large values of friction coefficient. There exist also some methods for contacts with or without frictions based on the latter technique (optimization technique), such as mathematical programming (Conry and Seireg, 1971; Haug et al., 1977; Hung and Saxcé, 1980) and formulations for linear complementarity problems (LCP) (Cottle et al., 1992; Anitescu and Potra, 1997; Meingast et al., 2014; Gholami et al., 2016). It has been proven that a properly formulated LCP always has a solution (Lotstedt, 1982). These techniques for contact constraints have been incorporated with some discrete

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models of numerical methods, such as boundary element method (BEM) (Eck et al., 1999), meshless methods (Li et al., 2007; Rabczuk and Belytschko, 2007; Rabczuk et al., 2010a, b; Rabczuk and Ren, 2017) and finite element method (FEM) (Papadopoulos and Solberg, 1998; Hild and Renard, 2010; Baillet and Sassi, 2002; Oden and Pires, 1984).

In these discrete numerical methods, FEM is an effective, stable and widely-used numerical method. Because of the over-stiff behavior of FEM models, the accuracy in stress solution of the method, especially using triangular elements, can often be low. To overcome the over-stiff issues, Liu et al. proposed weakened weak (W2) formulations of numerical methods based on the G space theory (Liu, 2010; Yue et al., 2016), such as the smoothed finite method (S-FEM) (Liu and Nguyen, 2010; Li et al., 2013, 2011) and smoothed point interpolation method (S-PIM) (Liu and Zhang, 2013).

Based on the standard finite element mesh of quadrilateral elements, a cell-based S-FEM (CS-FEM) (Liu et al., 2007; Liu et al., 2010; Thai-Hoang et al., 2011; Nguyen-Thoi et al., 2014) is introduced using a strain smoothing operations (Chen et al., 2015). In CS-FEM models, the smoothing domains (SDs)/smoothing cells (SCs) are created based on elements. Hence it is an S-FEM model closest to the FEM. Each element can be further subdivided into one or several quadrilateral smoothing cells. The properties of CS-FEM in solid mechanics have been studied from the analytical formulations and numerical examples, and some important properties can be found in (Liu et al., 2010). CS-FEM models are energy consistent and the energy solutions will change monotonously from the solution of CS-FEM with SC = 1 to that of CS-FEM with SC→∞. Especially, the CS-FEM model using 1SD for each element (CS-FEM-1SD) have the same properties with those of FEM using one Gauss-point when the same mesh is used; the solutions of CS-FEM models will be close to that of the standard FEM-Q4 model using four Gauss-points when SC approaches infinity.

The CS-FEM models are first used in the 2D static contact analysis based on the formulation of LCP. In this work, we firstly describe briefly a modified Coulomb friction contact model. Based on the gradient smoothing technique, the smoothed Galerkin weak-form with contact boundary is deduced. Then a set of discretized system of equations is constructed using CS-FEM models and constraints of contact point-pairs on contact interface and the discretized form for a modified coulomb contact is also given. These discretized equations are further converted into a formulation of LCP. Lastly, we present three numerical examples to validate the effectiveness of the method through comparison with the standard FEM-Q4.

2. Problem statement

Consider an elastic solid-solid contact problem with n_b bodies that may come into contact with each other, under the action of external loads. Assume that the configuration of the entire system Ω at time t is given and can be written as

$$\Omega = \cup_{i=1}^{n_b} \Omega_i, \tag{1}$$

where Ω_i represents the ith contact body bounded by $\Gamma_{\Omega_i} = \Gamma_u^{(i)} \cup \Gamma_\tau^{(i)} \cup \Gamma_c^{(i)}$, in which $\Gamma_u^{(i)}$ is the displacement boundary; $\Gamma_\tau^{(i)}$ is the traction boundary and $\Gamma_c^{(i)}$ is the contact boundary, as shown in Fig. 1.

2.1. Boundary value equations with contact

For solid body i, the incremental forms from time t to $t+\Delta t$ of its static equilibrium equation, the constitutive equation and compatibility equation are, respectively, written as,

$$\mathbf{L}^T \boldsymbol{\sigma}^{(i)} + \boldsymbol{b}^{(i)} = 0, \quad in\ \Omega^{(i)}, \tag{2}$$

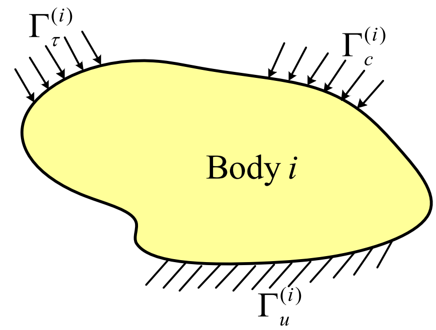

Fig. 1. Configuration of the ith contact body.

$$\boldsymbol{\sigma}^{(i)} = \mathbf{D}^{(i)} \boldsymbol{\varepsilon}^{(i)}, \tag{3}$$

$$\boldsymbol{\varepsilon}^{(i)} = \mathbf{L}\mathbf{u}^{(i)}, \tag{4}$$

where $\mathbf{L}$ is the matrix of differential operators. For 2D problems,

$$\mathbf{L} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix}, \tag{5}$$

and $\boldsymbol{b}^{(i)} = [b_x^{(i)} \quad b_y^{(i)}]^T$ and $\mathbf{u}^{(i)} = [u^{(i)} \quad v^{(i)}]^T$ are vectors of body force density increment and the displacement increment from time t to $t+\Delta t$ for body i. $\boldsymbol{\sigma}^{(i)} = [\sigma_{xx}^{(i)} \quad \sigma_{yy}^{(i)} \quad \tau_{xy}^{(i)}]^T$ and $\boldsymbol{\varepsilon}^{(i)} = [\varepsilon_{xx}^{(i)} \quad \varepsilon_{yy}^{(i)} \quad \gamma_{xy}^{(i)}]^T$ are vectors of the stress increment and strain increment, respectively. $\mathbf{D}^{(i)}$ is a matrix of elastic constants of the material.

The displacement increment, traction increment and contact traction increment boundary conditions are described, respectively, as follows:

$$\mathbf{u}^{(i)} = \bar{\mathbf{u}}^{(i)} \quad on\ \Gamma_u^{(i)}, \tag{6}$$

$$\mathbf{t}^{(i)} = \mathbf{L}_n^T \boldsymbol{\sigma}^{(i)} = \bar{\mathbf{t}}^{(i)} \quad on\ \Gamma_\tau^{(i)}, \tag{7}$$

$$\bar{\boldsymbol{\tau}}^{(i)} = \mathbf{L}_n^T \boldsymbol{\sigma}^{(i)} \quad on\ \Gamma_c^{(i)}, \tag{8}$$

where $\bar{\mathbf{u}}^{(i)}$ and $\bar{\mathbf{t}}^{(i)}$ are, respectively, given displacement increment at boundary $\Gamma_u^{(i)}$ and given traction increment at boundary $\Gamma_\tau^{(i)}$ in global coordinates; $\bar{\boldsymbol{\tau}}^{(i)}$ is a unknown contact traction increment on the contact boundary $\Gamma_c^{(i)}$ in global coordinates. $\mathbf{L}_n$ is a matrix composing of unit outward normal vectors n_x and n_y on boundary $\Gamma_\tau^{(i)}$ or $\Gamma_c^{(i)}$, and can be written as

$$\mathbf{L}_n = \begin{bmatrix} n_x & 0 & n_y \\ 0 & n_y & n_x \end{bmatrix}^T. \tag{9}$$

2.2. Modeling of contact interfaces

Consider a 2D elastic multi-body solid-solid contact system with modified Coulomb frictional contact, which is assumed as a surface-to-surface contact under the small deformation. The contact interface $\Gamma_{CI}^{(\alpha)}$ between two contact bodies I and II consists of contact surfaces $\Gamma_c^{(I)}$ and $\Gamma_c^{(II)}$, as shown in Fig. 2. The configuration of a contact point-pair ($\mathbf{x}^{(I)}$,$\mathbf{x}^{(II)}$) at time t on the contact interface $\Gamma_{CI}^{(\alpha)}$ is illustrated in Fig. 2a, in which $\mathbf{x}^{(I)}$ and $\mathbf{x}^{(II)}$ are, respectively, located on contact surfaces $\Gamma_c^{(I)}$ and $\Gamma_c^{(II)}$. In this paper, the contact point-pair strategy is nothing else that the well-known node-to-segment strategy, which consider a slave-node and its projection on the master surface. Fig. 2b illustrates that configuration of the contact point-pair ($\mathbf{x}^{(I)}$,$\mathbf{x}^{(II)}$) at time $t+\Delta t$ under the local