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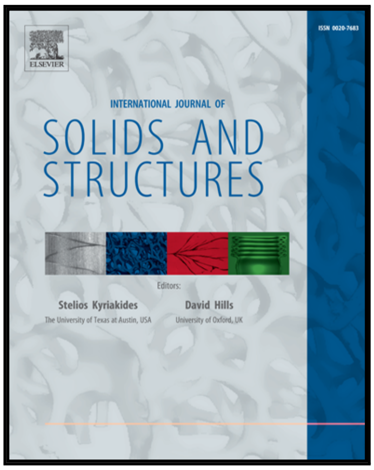

Application of optimal control method in buckling analysis of constrained elastica problems

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Application of optimal control method in buckling analysis of constrained elastica problems

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Abstract

Main concern of the paper is to illustrate the generic application of the optimal control method in the analysis of a constrained buckling problem and its simple and elegant formulation when compared to other techniques. Thus, an optimal control methodology is adopted to investigate the buckling behavior of thin elastic structures under the presence of unilateral constraints. We particularly explore the post-buckling response of a constant or variable length elastica constrained by rigid walls.

The equivalence between the calculus of variations and the optimal control is first demonstrated, with a main focus on the post-buckling behavior of a constant length elastica which is constrained by horizontal walls. The gradual construction of the constrained buckling problem as an optimal control problem from its equivalent Lagrangian form clearly shows its advantage when compared to the calculus of variations. The necessary optimality conditions, which constitute the Pontryagin's minimum or maximum principle, are also derived by considering a direct adjoining approach. The solution of the optimal control problem is then performed by applying a direct method.

Validation of the methodology is first achieved by reproducing examples available in the literature. Then the effects of factors, such as the geometry of the walls and the variability in the bending stiffness of the elastica, on its buckling response are analyzed. These constrained buckling problems are investigated for the first time, while through them it is also readily shown that the presence of geometric or material nonlinearity does not introduce any essential complexity in the buckling analysis when the optimal control methodology is adopted.

Keywords: Optimal control, constrained buckling, elastica, calculus of variations, unilateral constraints

1. Introduction

The buckling response of a thin elastic structure constrained by deformable or rigid walls is an essential problem in geo-engineering and medical applications. Examples involve the constrained buckling of drill-pipes in oil drilling operations (Wicks et al., 2008; Gao and Miska, 2009) and the insertion of a guidewire or a catheter into blood vessels (Tang et al., 2012; Lenoir et al., 2006). While the buckling of a slender object is a classical topic (Love, 2013; Wang et al., 2004), the constrained buckling problem has not been extensively investigated and two distinct classes of problems can be generally found; (1) the constrained buckling problem of constant length slender bodies (Domokos et al., 1997; Holmes et al., 1999; Feodosyev, 1977), and (2) the insertion buckling problem which involves variable length thin elastic structures (Denoel and Detournay, 2011; Huynen et al., 2016; Lu and Chen, 2008; Liakou and Detournay, 2017). The main objective of this paper is the development of an optimal control methodology that can be adopted for the investigation of a constrained buckling problem.

The general problem of an inextensible and unshearable elastic rod —the elastica— was first formulated as an isoperimetric problem by D. Bernoulli (1742) and it is considered a substantial contribution in the field of calculus of variations. This pioneering work was then extended by Euler (1744), who derived a complete list of physically feasible elastica figures. A comprehensive historical review on this subject can be found in (Levien, 2008). Nowadays, the buckling response of a constant length elastica under axial compression for Dirichlet or Neumann boundary conditions

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