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Analytical study of the dynamic behavior of a voided adhesively bonded lap joint under axial harmonic load

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ABSTRACT

In the present work an analytical model based on the improved shear-lag model was established for a bonded lap joint under a harmonic axial force, where a void is implanted in the overlap. The adherents were considered to be made from Aluminum while the adhesive was an epoxy with viscoelastic behavior. The model was validated using a 2D finite element model through ABAQUS software and the resonant axial frequencies where accurately predicted. The effect of the central void's size as well as the void's position and the loss factor of the adhesive on the modal behavior and also on the adhesive shear stress distribution and the level of the maximum adhesive stress was investigated.

1. Introduction

The light weight structure, time saving fabrication and good stress repartition over a wide area are the main reasons that have made from bonding an attractive method for the industrials to replace standard joining methods. Though, the mechanical characterization of such heterogeneous structures is constituting nowadays a hard challenge for many scientists and researchers. One of the most complex studies in this field are the analytical models due to the diversity of the materials, behaviors, geometries, conditions and assumptions residing in bonded assemblies.

One may find in the bibliography frequent number of analytical models for bonded assemblies subjected to static loadings. Some of them were based on lag models and classical stress and strain analyses (Adams et al., 1997; Adams and Mallick, 1992; Bigwood and Crocombe, 1989, 1990; Goland and Reissner, 1944; Hart-Smith, 1973; Tsai et al., 1998; Volkersen, 1938; Zou et al., 2004), others on energy formulations (Frostig et al., 1999; Nemes et al., 2006; Nemes and Lachaud, 2010) and a couple of models were based on multi-layer analysis (Diaz Diaz et al., 2009; Hadj-Ahmed et al., 2001; Yousefani and Tahani, 2013a, 2013b). Obviously the main target of those models was to determine the stress distribution within the adhesive layer and/or to predict the failure of the joint.

Moreover, the last two decades of the previous century were marked by few works devoted to the studies of free vibrations of bonded assemblies (He and Rao, 1992; Ko et al., 1995; Rao and He, 1992; Rao and Zhou, 1994; Saito and Taini, 1984). However, the solution of such models has needed huge computational memory and time consuming

However, when moving to bonded joints subjected to dynamic loadings, the bibliography does not offer this large number of analytical models. One of the first ever models under harmonic load was the one of Vaziri et al. (2001) who established a closed form analytical model on a voided single lap joint subjected to a harmonic peeling load; they studied the effect of the void's size and position on the fundamental frequency and on the peeling and shear stresses in the adhesive, the adhesive behavior was viscoelastic however the classical shear lap approach was applied where it was assumed that the shear stress through the thickness of the adherents is constant. This work was repeated (Vaziri and Nayeb-Hashemi, 2002) for voided tubular joints subjected to harmonic axial loading. Sato (2009) applied the Laplace transform to find the shear stress only at the joint edge of a lap strap joint with infinite length this under impulse and indicial input normal stresses; he applied the classical shear lag model. This work was improved (Hazimeh et al., 2015) when the modified shear lag approach was applied (same as Tsai et al., 1998); this approach assumes a linear distribution of the shear stress through the adherents thicknesses; this approach is closer to the real behavior of the adherents; in this work, a double lap geometry and an elastic adhesive were considered. The double lap joint was also studied analytically by Challita and Othman (2012) who considered just the bonded region with an elastic adhesive but with the modified shear lag approach; an axial harmonic load was applied at the middle plate. A critical frequency was defined and many dimensionless parameters to evaluate the maximum shear stress and the shear stress homogeneity were established. The model was vali-

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Nomenclature

Symbol	Designation
t	time
ω	angular frequency of the harmonic load
x	horizontal coordinate (along the length)
y	vertical coordinate (along the thickness)
$\tilde{\mathbf{q}}$	complex magnitude of the quantity $q = \mathrm{Re}(\tilde{q}.\exp(i\omega t))$
$\mathbf{Q}_j^{(n)}$	quantity related to substrate $n = 1;2$ in region $j = $ I; II; III; IV; V
t_s	thickness of the substrates
t_a	thickness of adhesive layer
a	length of non-bonded substrates
b	length of overlap portion at the left of the void
c	length of the void
d	length of overlap portion at the right of the void
E_s	Young's modulus of the substrates
G_s	shear modulus of the substrates
ρ_s	density of the substrates
G_a	shear modulus of the adhesive
η	loss factor of the adhesive
$G_a{}^*$	complex Shear modulus of the adhesive $= G_a (1 + i.\eta)$
u	axial displacement of the substrates
u_i	axial displacement at the interface adhesive-substrate
$<u>$	through-thickness average axial displacement of the substrates
τ	shear stress in the substrates
τ_a	shear stress in the adhesive layer
N	axial force per unit width applied on the substrates
P_0	external force per unit width applied at the right end of the lower substrate
χ	normalized void size to the overlap length
δ	normalized void position measured from left end of the overlap

Constants

$\mu^* = \frac{2.G_a{}^*}{t_s.t_a.E_s}$; $\lambda^* = 1 + \frac{2.t_s.G_a{}^*}{3.t_a.G_s}$; $\xi^{*2} = \frac{\rho_s.\omega^2}{E_s}$; $\zeta^{*2} = \frac{\mu^*}{\lambda^*} - \xi^{*2}$

dated numerically. Almitani and Othman (2016) have repeated the previous analytical model considering viscoelastic adherents and adhesive: they examined the effect of adhesive loss factor, adhesive and adherent materials on the resonant frequencies. Challita and Othman (2017) have extended this latter work by considering the whole assembly and not only the bonded zone.

The actual analytical model considers that a void is included in a lap joint that is a half symmetric double lap joint subjected to an axial harmonic load. The analytical transfer function that leads to the determination of the resonant frequencies was validated numerically. Hereafter, the effect of the void's size and position on the axial modal behavior and adhesive shear stress was analyzed.

2. Analytical model

The adhesively bonded half double lap joint containing a void in the overlap is shown in Fig. 1. It consists of two identical rectangular substrates, the lower indexed by "1" and the upper by "2" bonded together through a voided adhesive layer. The left end of the upper adherent is cantilevered while the right end of the lower adherent is subjected to a harmonic axial force $P_0 = \mathrm{Re}(\tilde{P}_0.\exp(i\omega t))$. The assembly is divided into five regions upon the configuration change along the x direction. These regions are numbered from I to V and shown also on Fig. 1. The free body diagrams on a differential length dx of the substrates in each region is drawn in Fig. 2(a)–(e) corresponding to regions I–V respectively.

This model, aiming at determining the harmonic response, is based on the improved shear lag model stated in (Tsai et al., 1998) in static loading and in (Almitani and Othman, 2016; Challita and Othman, 2017) in dynamic loading; it considers a linear distribution of the shear stress through the thickness of each substrate; this is illustrated in the graph of Fig. 3. This model is developed under the assumptions of a longitudinal motion of the system and a constant through-thickness adhesive shear stress.

It should be noticed that, to avoid repetition of many calculation details, only the main steps will be mentioned in the coming lines. For more calculation details, refer to the work of Almitani and Othman, (2016).

2.1. Region I

According to the free body diagram in Fig. 2(a), the Newton's second law applied on the upper substrate in this region yields to the following equation:

$$\frac{d\tilde{N}_I^{(2)}(x,\omega)}{dx} = -\rho_s t_s \omega^2 \tilde{u}_I^{(2)}(x,\omega) \tag{1}$$

On the other hand, applying Hooke's law under axial stress, then multiplying by the adherent thickness to switch to the axial force per unit width, then differentiating once with respect to x, one obtains the following equation:

$$\frac{d\tilde{N}_I^{(2)}(x,\omega)}{dx} = E_s t_s \cdot \frac{d^2\tilde{u}_I^{(2)}(x,\omega)}{dx^2} \tag{2}$$

Equating left members in Eqs. (1) and (2) a second order differential equation in terms of the axial displacement will be obtained. The solution of this equation gives the expression of the axial displacement. From Hooke's law (Eq. (2) before differentiation), the expression of the axial effort is also determined:

$$\tilde{u}_I^{(2)}(x,\omega) = A_I(\omega)\cdot\exp(-i\xi^* x) + B_I(\omega)\cdot\exp(+i\xi^* x) \tag{3}$$

$$\tilde{N}_I^{(2)}(x,\omega) = i\xi^* E_s t_s[-A_I(\omega)\cdot\exp(-i\xi^* x) + B_I(\omega)\cdot\exp(+i\xi^* x)] \tag{4}$$

$A_I(\omega)$ and $B_I(\omega)$ are two propagating damped waves and mathematically constants of integration generated by solving a second order differential equation.

2.2. Region II

This region contains a portion of the overlap. Equations will be written for both substrates and also for the adhesive layer. As mentioned earlier, the improved shear lag model is represented graphically in Fig. 3. Mathematically, this model could be written as follows:

$$\tilde{\tau}_{II}^{(1)}(x,y,\omega) = \frac{y}{t_s}\cdot\ \tilde{\tau}_a(x,\omega) \tag{5a}$$

$$\tilde{\tau}_{II}^{(2)}(x,y,\omega) = \left(1 - \frac{y-(t_s+t_a)}{t_s}\right)\cdot\ \tilde{\tau}_a(x,\omega) \tag{5b}$$

Applying the classical Hooke's law in shear on each adherent, and replacing the shear strain by the differentiation of the axial displacement with respect to the thickness coordinate y, then integrating for the displacement along y, one gets Eqs. (6a) and (6b) of the axial displacements of each substrate:

$$\tilde{u}_{II}^{(1)}(x,y,\omega) = \tilde{u}_{iII}^{(1)}(x,\omega) + \frac{y^2 - t_s^2}{2t_s G_s}\cdot\ \tilde{\tau}_a(x,\omega) \tag{6a}$$