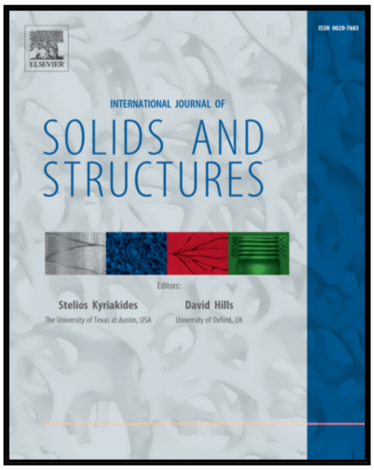

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Three-dimensional stress analysis for beam-like structures using Serendipity Lagrange shape functions

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Abstract

Simple analytical and finite element models are widely employed by practising engineers for the stress analysis of beam structures, because of their simplicity and acceptable levels of accuracy. However, the validity of these models is limited by assumptions of material heterogeneity, geometric dimensions and slenderness, and by Saint-Venant's Principle, i.e. they are only applicable to regions remote from boundary constraints, discontinuities and points of load application. To predict accurate stress fields in these locations, computationally expensive three-dimensional (3D) finite element analyses are routinely performed. Alternatively, displacement based high-order beam models are often employed to capture localised three-dimensional stress fields analytically. Herein, a novel approach for the analysis of beam-like structures is presented. The approach is based on the Unified Formulation by Carrera and co-workers, and is able to recover complex, 3D stress fields in a computationally efficient manner. As a novelty, purposely adapted, hierarchical polynomials are used to define cross-sectional displacements. Due to the nature of their properties with respect to computational nodes, these functions are known as Serendipity Lagrange polynomials. This new cross-sectional expansion model is benchmarked against traditional finite elements and other implementations of the Unified Formulation by means of static analyses of beams with different complex cross-sections. It is shown that Serendipity Lagrange elements solve some of the shortcomings of the most commonly used Unified Formulation beam models based on Taylor and Lagrange expansion functions. Furthermore, significant computational efficiency gains over 3D finite elements are achieved for similar levels of accuracy.

Keywords: Finite elements, unified formulation, 3D stress fields, T section

1. Introduction

In engineering design, long slender structures are typically analysed using axiomatic beam models. These models are valid under the premise that the longitudinal dimension of a structure is at least one order of magnitude larger than representative cross-sectional dimensions. This geometric feature allows the governing elasticity equations to be reduced from three to one dimension, (with the reference axis coinciding with the beam axis), and in so doing, brings about significant physical insight and computational benefits. The aim of this current work is to build a model capable of capturing three-dimensional stress fields for beam-like structures in a computationally efficient manner. In this regard, many efforts have been carried out in the last few decades. A brief historical excursus is now presented.

Classical axiomatic theories are sufficiently accurate for relatively slender beam structures (length to thickness ratio $L/t > 20$) but their accuracy is limited by Saint-Venant's principle, i.e. to regions remote

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