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Anisotropic nonlocal damage model for materials with intrinsic transverse isotropy

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ABSTRACT

This paper presents the theoretical formulation and numerical implementation of an anisotropic damage model for materials with intrinsic transverse isotropy. Crack initiation and propagation are modeled by phenomenological damage evolution laws, controlled by four equivalent strain measures. The latter are constructed so as to distinguish the mechanical response of the material in tension and compression, along the direction perpendicular to the bedding plane and within the bedding plane. To avoid mesh dependency induced by softening, equivalent strains are replaced by nonlocal counterparts, defined as weighted averages over a neighborhood scaled by two internal length parameters. Finite Element equations are solved with a normal plane arc length control algorithm, which allows passing limit points in case of snap back or snap through. The model is calibrated against triaxial compression tests performed on shale, for different confinements and loading orientations relative to the bedding plane. Gauss point simulations confirm that the model successfully captures the variation of uniaxial tensile strength with respect to the bedding orientation. Finite Element simulations of three-point bending tests and compression splitting tests show that nonlocal enhancement indeed avoids mesh dependency, and that the axial and transverse dimensions of the damage process zone are scaled by the two characteristic lengths. Results further show that the damage process zone is direction dependent both in tension and compression. The model can be used for any type of textured brittle material; it allows representing several concurrent damage mechanisms in the macroscopic response and interpreting the failure mechanisms that control the damage process zone.

1. Introduction

Many geomaterials exhibit strong orientation dependent mechanical behavior (anisotropy) due to bedding, layering or crack patterns, as evidenced in shale (Niandou et al., 1997; Gautam and Wong, 2006; Sone and Zoback, 2013), clay stone (OKA et al., 2002), schists (Nasseri et al., 2003) and sand (Dafalias et al., 2004). Laboratory tests, such as uniaxial and triaxial compression tests (Niandou et al., 1997; Nasseri et al., 2003; Fu et al., 2012), Brazilian indirect tension tests (Cho et al., 2012; Vervoort et al., 2014), direct shear tests (Heng et al., 2015) and triaxial shear tests (Ye and Ghassemi, 2016), further demonstrate that material strength and failure modes significantly depend on the confining pressure and the loading orientation with respect to microstructure. Prior to crack propagation, most geomaterials can be considered transverse isotropic: the maximum uniaxial compressive strength is reached when weak planes are either parallel or perpendicular to the loading direction, and the minimum strength is reached when weak planes are oriented at $30^\circ - 60^\circ$ with respect to the loading direction (Donath, 1961; Niandou et al., 1997). In indirect tensile tests, the tensile strength is maximum when tensile stress is applied within the weak plane, and gradually decreases as the orientation angle between the tensile stress direction and the bedding plane increases (Cho et al., 2012).

In geomaterials models, intrinsic and induced anisotropy are either accounted for in the failure criterion or in the expression of the free energy. Hill (1948) extended the von Mises yield criterion to orthotropic ductile materials, by using 6 quadratic stress terms. To further account for the strength difference in tension and compression, Hoffman (1967) added 3 linear terms of stress into Hill's failure criterion. Tsai and Wu (1971) then expressed failure criteria that depend on all possible linear and quadratic stress terms. These yield criteria were used by de Borst (Schellekens and De Borst, 1990; Hashagen and De Borst, 2001) to model perfectly plastic and hardening materials. Reinicke and Ralston (1977) carried out limit analyses using parabolic yield functions (Hoffman's criteria).

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Other approaches were proposed to introduce either the fourth order projection tensor or the second order microstructure tensor in the yield criteria or in the expression of the free energy. Boehler and Sawczuk (1977) used the microstructure tensor in the constitutive model for transverse isotropic materials. Cazacu et al. (1998) and Cazacu and Cristescu (1999) employed a fourth order projection tensor to transform the stress tensor into a characteristic tensor with embedded microstructure information. They extended the Mises–Schleicher yield criterion initially expressed for isotropic materials to transversely isotropic materials, by using the fourth order characteristic tensor. Another approach, based on different projection tensors, consists in projecting the strength in Drucker–Prager and Mohr–Coulomb yield criteria (Rouabhi et al., 2007). The microstructure tensor can also be constructed with eigenvectors representing the axes of material symmetry to capture the orientation dependence of strain hardening, softening, damage and plasticity in shale (Pietruszczak and Mroz, 2000; 2001; Pietruszczak et al., 2002; Chen et al., 2010). For soils, fabric tensors were used to represent microstructure in yield criteria (Oda and Nakayama, 1989). Thermodynamic models were also proposed, in which the free energy was expressed in terms of microstructure tensor and strain invariants (Halm et al., 2002; Nedjar, 2016).

Alternatively, the Representative Element Volume (REV) can be viewed as a set of cracks or planes of discontinuities. Intrinsic anisotropy is accounted for by assigning different material properties to crack families of different orientations (Chen et al., 2012; Hu et al., 2013). In micromechanics models, a static constraint is applied to projections of stress on the crack planes, and the expression of the REV free energy is obtained by homogenization. In microplane models, a kinematic constraint is applied to projections of strains on the crack planes; the principle of virtual work is used to upscale the microscopic relationships and calculate the macroscopic stress. Anisotropy is accounted for by considering complex microplane orientation distributions and by formulating different evolution criteria for different microplanes (Li et al., 2017).

Once implemented in the Finite Element Method (FEM), anisotropic models that account for plastic/damage softening suffer from mesh dependency. Integration-based nonlocal formulations alleviate spurious mesh sensitivity but cannot be easily used with stress-based yield/damage criteria, in which the out-of-balance stress at a point has to be calculated iteratively from the yet-unknown stress in a given neighborhood. Hence in this paper, we integrate a measure of strain to formulate a nonlocal anisotropic damage model for transverse isotropic geomaterials. In Section 2, we formulate a constitutive law for predicting stress-induced anisotropy in an initially transverse isotropic material. The evolution laws of damage components are expressed in terms of equivalent strains, which are direction dependent. Two internal length parameters are used to avoid mesh dependency and account for intrinsic anisotropy. In Section 3, we calibrate the model against stress/strain curves obtained during triaxial compression tests performed on shale. A sensitivity analysis is presented based on a series of uniaxial tension tests simulated on a single element (at the Gauss point). In Section 4, we present simulations of three-point bending and splitting tests and we show that the size of the fracture process zone is direction dependent, but mesh independent. Results also reveal the underlying failure mechanisms associated to damage orientation. Note that we use Voigt matrix notations throughout the paper. Lower cases are used for scalar variables, bold lower cases for vectors and bold upper cases for matrices. Note that the soil mechanics sign convention is adopted throughout the paper, in which compression is counted positive.

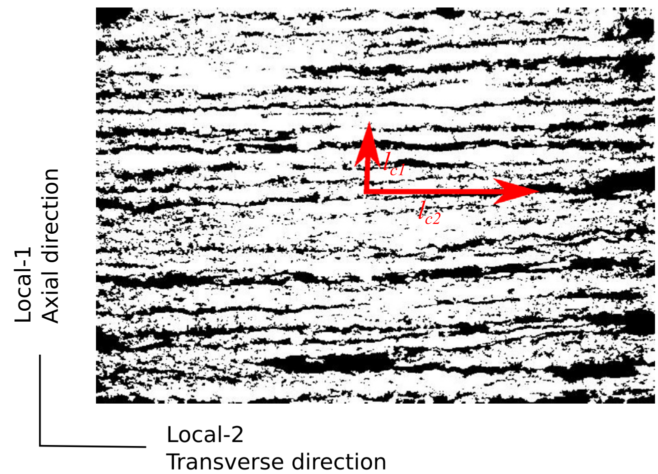

Fig. 1. Definition of the intrinsic damage directions in transverse isotropic shale, modified from Bramlette (1943).

2. Formulation and implementation of the nonlocal anisotropic damage model for transverse isotropic materials

2.1. Damage operator and damaged stiffness tensor

At the scale of the REV, the inception, propagation and coalescence of micro-cracks result in hardening or softening of stress/strain relations and stiffness reduction. The nominal stress, $\boldsymbol{\sigma}$, is related to the damaged effective stress, $\hat{\boldsymbol{\sigma}}$, through

$$\hat{\boldsymbol{\sigma}} = \mathbf{M}\boldsymbol{\sigma} \tag{1}$$

Where $\mathbf{M}$ is a damage operator. Assuming that damage components in each direction evolve independently, the damage operator $\mathbf{M}$ has a diagonal form, as follows:

$$M_{ii} = \frac{1}{1-\omega_i} \quad i = 1, 2, \ldots, 6 \tag{2}$$

Note that Voigt notations are adopted here, so that $\hat{\sigma}_4 = \hat{\tau}_{23} = \frac{\tau_{23}}{1-\omega_4}$, in which:

$$\begin{aligned} \omega_4 &= 1-(1-\omega_2)(1-\omega_3) \\ \omega_5 &= 1-(1-\omega_1)(1-\omega_3) \\ \omega_6 &= 1-(1-\omega_1)(1-\omega_2) \end{aligned} \tag{3}$$

The diagonal form of $\mathbf{M}$ ensures that the damaged compliance matrix resulting from Eq. (1) is symmetrical. We consider a geomaterial with transverse isotropy with respect to the normal direction of bedding planes. Fig. 1 shows the example of shale, which is a sedimentary rock (Gautam and Wong, 2006; Waters et al., 2011; Ye et al., 2016). We set the local coordinate system so that direction 1, called the axial direction, is perpendicular to the bedding plane. Directions 2 and 3, along the bedding plane, are called transverse directions. Correspondingly, in Eq. (2), ω_1 is called axial damage and ω_2, ω_3 are the transverse damage variables.

We focus on transverse isotropic behavior in quasi-brittle materials. With negligible inelastic deformation, the non-linear stress/strain behavior results from damage evolution only (micro-crack development). Adopting the principle of strain equivalence, the constitutive relation is expressed as

$$\boldsymbol{\epsilon} = \mathbf{S}^0\mathbf{M}\boldsymbol{\sigma}. \tag{4}$$

For a transverse isotropic material, the elastic compliance matrix $\mathbf{S}^0$ depends on 5 parameters. In the local coordinate system, $\mathbf{S}^0$ is

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