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Design of ultra-thin shell structures in the stochastic post-buckling range using Bayesian machine learning and optimization

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ABSTRACT

A data-driven computational framework combining Bayesian regression for imperfection-sensitive quantities of interest, uncertainty quantification and multi-objective optimization is developed for the design of complex structures. The framework is used to design ultra-thin carbon fiber deployable shells subjected to two bending conditions. Significant increases in the ultimate buckling loads are shown to be possible, with potential gains on the order of 100% as compared to a previously proposed design. The key to this result is the existence of a large load reserve capability after the initial bifurcation point and well into the post-buckling range that can be effectively explored by the data-driven approach. The computational strategy here presented is general and can be applied to different problems in structural and materials design, with the potential of finding relevant designs within high-dimensional spaces.

1. Introduction

The recent resurgence of interest in buckling of slender structures (Hu and Burgueño, 2015; Reis, 2015) can be largely explained by the advent of new manufacturing techniques (Kalpakjian et al., 2014) for complex shapes, as well as improved modeling capabilities (Benson et al., 2010; Bai et al., 2015) that leverage the extensive theoretical understanding of buckling (Hutchinson and Koiter, 1970; Bažant and Cedolin, 2010). These developments have spawned a myriad of creative solutions for a wide range of applications such as energy harvesters (Chen et al., 2010; Wang et al., 2014), sensors (Elvin et al., 2006; Kiyono et al., 2012), dampers and absorbers (Dong and Lakes, 2013; Kim et al., 2013), actuators (Loukaides et al., 2014; Lazarus and Reis, 2015), morphing structures (Diaconu et al., 2008; Daynes et al., 2014), and deployable structures (Pellegrino, 2014; Mallikarachchi and Pellegrino, 2014), all of which exploit the geometrically non-linear behavior of thin shell structures.

These structures are designed to capture the benefits of the first bifurcation point (initial buckling). In these cases, post-buckling is mostly viewed as a sudden behavior that leads to large configuration changes, often occurring as a form of snap-through or to a lesser extent snap-back. Hence, post-buckling behavior and subsequent bifurcations are not usually a design target (Hu and Burgueño, 2015).

However, there are structures where the post-buckling behavior assumes particular importance. Thin-walled structures provide some of the most relevant examples, as can be seen in studies showing the effects of cutouts in composite shells (Tafreshi, 2002) or of nonuniform thickness in steel shells (Aghajari et al., 2006), as well as investigations on functionally graded carbon nanotube-reinforced shells undergoing thermal post-buckling (Shen, 2012) and functionally graded shallow plates (Woo et al., 2005). Recently, Leclerc et al. (2017) noted that an ultra-thin composite Triangular Rollable And Collapsible (TRAC) boom is able to carry significantly increased loads well into the post-buckling regime. The present study focuses on designing TRAC booms to improve their buckling and post-buckling behavior through a data-driven computational framework.

In data-driven approaches (Bisagni and Lanzi, 2002; Yvonnet and He, 2007; Ning and Pellegrino, 2015; Bessa et al., 2017) a new model or design is found by collecting enough data about the response of the structure or material under multiple input conditions. In principle, data can be collected by experimental testing, analytical or computational predictions. Yet, in most engineering applications experimental characterization of previously untested designs is too time-consuming to gather enough data in a timely manner, and most applications are too complex to be predicted by probabilistic analytical models (Elishakoff, 2014). Hence, computational predictions are often the only viable resource to explore the

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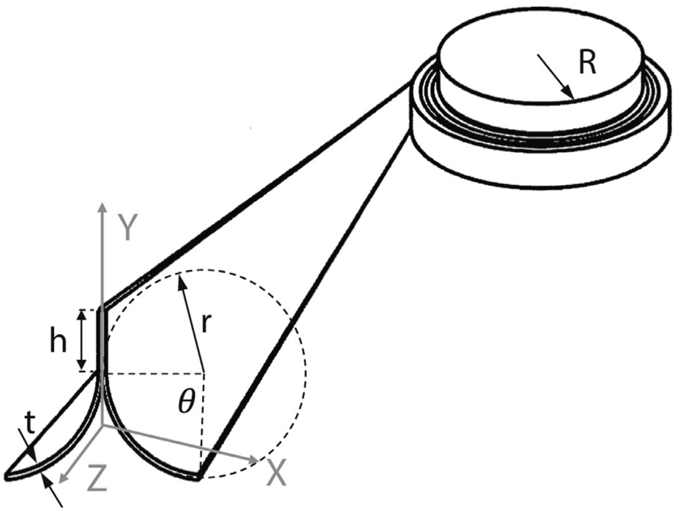

Fig. 1. Schematic of a TRAC boom partially coiled on a spool (modified from Murphey and Banik, 2011).

design space and generate enough data to use machine learning and/or optimization.

Multiple authors have been exploring the use of data-driven approaches for different scientific and engineering applications. Notable examples exist in computer science with artificial intelligence algorithms that master the game of GO (Silver et al., 2016), materials science with data mining of first principle calculations leading to discoveries of new material compounds (Curtarolo et al., 2003; Fischer et al., 2006; Saal et al., 2013; Gautier et al., 2015), fluid mechanics in the characterization of flows with high Reynolds numbers (Ling and Templeton, 2015), and different solid mechanics applications (Bisagni and Lanzi, 2002; Yvonnet and He, 2007; Bessa et al., 2017).

This article extends a recently developed data-driven framework for materials and structures (Bessa et al., 2017) to the design of optimized structures with uncertain response. The proposed approach is illustrated for the design of TRAC booms, in order to increase their ultimate buckling limit, but it can be applied to any other structure or material. The extended framework includes two significant contributions: 1) machine learning for noisy observations with uncertainty quantification; and 2) introducing a multi-objective optimization step after the machine learning procedure to determine optimal designs. The first extension is crucial for design and analysis of imperfection-sensitive structures (with noisy or uncertain response). The second extension is relevant when the goal is not just establishing the relationship between input design descriptors and output performance of the structure (or material), but also to find the set of input descriptors that leads to an optimal response within given constraints.

The paper is laid out as follows. Section 2 presents the ultra-thin composite TRAC boom structure to which the data-driven framework is applied. Section 3 discusses the data-driven framework for noiseless applications in 3.1, and for noisy applications with multi-objective optimization goals in 3.2. Concluding remarks are included in Section 4.

2. Behavior of ultra-thin TRAC booms

TRAC booms were first proposed and developed by Murphey and Banik (2011). Fig. 1 shows a schematic of a TRAC boom, partially coiled on a spool. This type of structure consists of two thin cylindrical shells of uniform thickness, t, whose uniform cross-section consists, in the deployed, i.e. unstressed configuration, of a circular arc of radius r subtending an angle θ, and a straight segment of length h. The two shells are mirror-symmetric and the flat parts are bonded together in the unstressed, i.e. longitudinally straight, configuration. Thus, the deployed structure has an approximately triangular cross-section, mirror-symmetric with respect to the plane $y-z$, consisting of single-thickness *transversely curved flanges* and a double-thickness *flat web*.

This structure can be packaged by flattening the cross-section and longitudinally coiling the boom on a spool of radius R, as shown in Fig. 1. The coiling behavior resembles the classical tape-spring coiling (Seffen and Pellegrino, 1999), but with some additional complications that are the focus of current research (Murphey et al., 2017; Leclerc et al., 2018).

The deployed geometry of the TRAC boom is then fully characterized by the boom length L and the cross-section parameters: web height h (with thickness $2t$), flange radius r, angle θ (with thickness t).

The motivation for the present study comes from the use of TRAC booms as structural components of future spacecraft. A particular application that is under investigation (Leclerc et al., 2017) uses TRAC booms suspended from prestressed cables to support lightweight solar arrays. In this type of application the solar radiation pressure loading on the solar cells causes significant bending moments in the TRAC booms. These loads can cause premature buckling of the TRAC booms, due to their extremely small thickness – see Fig. 2. Note that, since initial buckling happens so prematurely, the postbuckling response should be considered when designing structures of this kind. This further complicates the design of this imperfection sensitive ultra-thin structure, motivating the developments introduced in this article.

Fig. 2 shows the predictions obtained from nonlinear finite element analyses using the arc-length method to determine the postbuckling response of an ultra-thin composite TRAC boom studied by Leclerc et al. (2017). The structure was subjected to two separate boundary conditions: (a) bending moment around X leading to compression at the outer edge of the web; and (b) bending moment around Y leading to the formation of a kink in the compressed flange – see Fig. 2a and b, respectively. The same nominal geometric parameters reported by Leclerc et al. (2017) are used herein: total length $L = 504$ mm, and cross-section parameters $r = 10.6$ mm, $\theta = 105^{\circ}$, and $h = 8$ mm. The material is a composite laminate with stacking sequence $[0^{\circ}, 90^{\circ}]_S$ and nominal post-cure thickness of $t = 71\,\mu$m, where the four composite plies are stacked from a 17GSM unidirectional tape supplied by North Thin Ply Technology (T800 fibers and ThinPreg 120EPHTg-402 epoxy resin). The orthotropic elastic properties of each ply are considered as $E_1 = 128.0$ GPa, $E_2 = 6.5$ GPa, $\nu_{12} = 0.35$, $G_{12} = G_{13} = G_{23} = 7.5$ GPa.

The dashed lines in Fig. 2c and d show the responses predicted for the idealized geometry, where a negligibly small imperfection based on the first buckling mode was seeded to numerically resolve the first bifurcation point. Details about the six imperfect cases shown in the figure are discussed later in Appendix A. At this point, it is important to note that the first bifurcation point occurs prematurely for both loading conditions due to the extreme thinness of the structure, but there is a significant residual strength until the ultimate buckling limit of the structure is reached. Here the *ultimate buckling limit* is defined as the analytical maximum in the moment-angle response of the structure. Due to the complexity and stochasticity of the buckling and postbuckling behavior of the TRAC boom, the particular geometry considered in Fig. 2 is likely not optimal for given quantities of interest, e.g. maximizing both buckling limits. Since closed form solutions to find such optimal geometries do not exist, the viable alternative is to use a data-driven approach applicable to stochastic responses.

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