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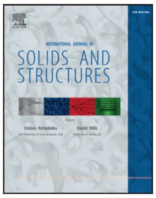

A novel positive/negative projection in energy norm for the damage modeling of quasi-brittle solids

Jian-Ying Wu[a,*], Miguel Cervera[b]

[a] State Key Laboratory of Subtropical Buildig Science, South China University of Technology, Guangzhou 510641, China
[b] CIMNE, Technical University of Catalonia, Edificio C1, Campus Norte, Jordi Girona 1–3, Barcelona 08034, Spain

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ABSTRACT

The asymmetric tensile/compressive material behavior and microcracks closure-reopening (MCR) effects exhibited by quasi-brittle solids are of significant importance to the nonlinear responses of engineering structures under cyclic loading, e.g., earthquake excitations. Based on our previous work (Cervera et al., 1995; Faria et al., 1998; Wu et al., 2006) this work addresses a novel thermodynamically consistent unilateral damage model for concrete. In particular, the positive/negative projection (PNP) of the effective stress tensor and the additive bi-scalar damage constitutive relation are maintained owing to the conceptual simplicity and computational efficiency. It is found that the classical PNP widely adopted in the literature is not optimal for this damage model, since the resulting stiffness is not always of major symmetry. Consequently, a well-defined free energy potential does not exist in general cases and the model cannot be cast into the framework of thermodynamics with internal variables. Furthermore, the damage induced anisotropy cannot be captured, exhibiting excessive lateral deformations under uniaxial tension. To overcome the above issues, a novel PNP, variationally interpreted as the closest point projection of the effective stress in energy norm, is proposed with closed-form solution. With the novel PNP, the secant stiffness tensor of the proposed unilateral damage model always possesses major symmetry and exhibits orthotropic behavior under uniaxial tension and mixed tension/compression. The corresponding thermodynamics framework is then given, resulting in an energy release rate based rounded-Rankine type damage criterion appropriate for tensile failure in quasi-brittle solids. Several numerical examples of single-point verifications and benchmark tests are presented. It is demonstrated that the proposed model is capable of characterizing localized failure of concrete under proportional and non-proportional static loading, as well as the MCR effects under seismic cyclic loading.

1. Introduction

Failure of quasi-brittle solids, e.g., concrete, rock, ceramics, etc., is mainly dominated by microcracks evolution on the mesoscopic scale. On the one hand, they are prone to localized failure in dominant tension, and exhibit rather inferior tensile behavior and stiffness degradation compared to the relatively high strength and ductile performances in compression. On the other hand, microcracks tend to close upon load reversal such that the material stiffness can be largely recovered (Reinhardt and Cornelissen, 1984; Mazars et al., 1990), known as the so-called unilateral effects. The asymmetric tensile/compressive material behavior and microcracks closure-reopening (MCR) effects are of significant importance to the responses and safety of engineering structures under static and seismic loadings.

Owing to the pioneering work of Kachanov (1958), continuum damage mechanics (CDM) has nowadays been widely adopted in the constitutive modeling of concrete like quasi-brittle solids; see the monographs (Krajcinovic, 2003; Lemaitre and Desmorat, 2005; Murakami, 2012) and the references therein. Restricting our attention to concrete under cyclic loading, a large volume of damage models have been proposed in the literature; see Mazars and Pijaudier-Cabot (1989), Cervera et al. (1995), Faria et al. (1998), Lee and Fenves (1998), Comi and Perego (2001), Wu et al. (2006), Abu Al-Rub and Kim (2010), Miehe et al. (2010a), Grassl et al. (2013) among many others. In most of these damage models, two scalar internal variables, say, i.e., d^+ and d^-, both in the range [0,1], are adopted to characterize the asymmetric tensile/compressive material behavior, respectively. With this strategy,

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[*] Corresponding author.
E-mail address: jywu@scut.edu.cn (J.-Y. Wu).

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the MCR effects upon load reversals can also be modeled straightforwardly by (partially) activating/deactivating the corresponding damage variable dependent on the specific stress/strain state.

To the above end, it is necessary to discriminate the dominant tension (positive) and compression (negative) states in general 3-D cases. In the 1980s the French scholars first decomposed the stress tensor into its positive and negative components based on the spectral decomposition; see Mazars and Pijaudier-Cabot (1989) for the review. During this stage, another celebrated work is Ortiz (1985) in which the fourth-order positive/negative projection (PNP) operators of the stress tensors were first introduced explicitly in the damage constitutive relation. Since then, in the damage modeling of concrete the PNP of a specific second-order tensor, e.g., the strain (Simó and Ju, 1987; Ju, 1989), the nominal stress (Yazdani and Schreyer, 1990; Hansen and Schreyer, 1994, 1995; Lubarda et al., 1994) and the effective stress (Cervera et al., 1995; Faria et al., 1998; Wu et al., 2006; Abu Al-Rub and Kim, 2010; Grassl et al., 2013), etc., has been the standard strategy. Besides the expressions given in Ortiz (1985), two alternative fourth-order PNP operators were also proposed (Simó and Ju, 1987; Carol and Willam, 1996). In the last reference (Carol and Willam, 1996), different PNP operators for the damage modeling of the MCR effects were systematically investigated. It was found that for a generic anisotropic damage model, all the existing PNP operators cannot guaranteed zero energy dissipation during a closed loading cycle, violating the second law of thermodynamics. Recently, this issue was revisited by the author (Wu and Xu, 2013). Unified expressions for the classical fourth-order PNP operators were established with all the previous ones included as its particular examples. Furthermore, the thermodynamical consistent PNP operators were derived for the first time, completely removing the aforesaid issue.

Among the large quantities of damage models for concrete, the one proposed by Cervera et al. (1995) and Faria et al. (1998) deserves further comments. In this model the PNP of the effective stress was first adopted for modeling the asymmetric tensile/compressive material behavior and the MCR effects of concrete. This strategy results in a theoretically simple additive bi-scalar damage constitutive relation. In particular, the plastic strains can be straightforwardly incorporated by the effective stress space plasticity (Ju, 1989) as in the work of Wu et al. (2006). Furthermore, as the effective stress tensor is defined in the context of strain equivalence (Simó and Ju, 1987), the strain-driven numerical algorithm (Faria et al., 1998; Wu et al., 2006) is very robust and efficient, particularly useful for the application to practical engineering structures (Wu and Li, 2007). Owing to the theoretical and numerical advantages, the additive bi-scalar damage constitutive relation has been widely adopted in later development of concrete models (Li and Ren, 2009; Abu Al-Rub and Kim, 2010; Pelà et al., 2011; Gernay et al., 2013; Grassl et al., 2013). However, for the classical PNP scheme considered in the literature, this additive bi-scalar damage model degenerates in the uniaxial stress state to an isotropic one which is known being unable to capture the damage induced anisotropy. More importantly, the corresponding stiffness tensor is not of major symmetry in the mixed tension/compression states. That is, it is impossible to define a *unique* free energy potential function. Accordingly, the original additive bi-scalar damage model cannot be cast into the framework of thermodynamics with internal variables. This is in strong contrast to the bi-scalar damage models based on the positive/negative split of the stress or strain tensor (Mazars and Pijaudier-Cabot, 1989; Ortiz, 1985; Miehe et al., 2010a) which are thermodynamically consistent. Last but not the lest, another side effect is that the damage criteria can only be postulated in an *ad hoc* or heuristically manner.

In order to remedy the aforesaid issues, Cervera and Tesei (2017) recently proposed a *multiplicative* bi-scalar damage model which is distinct from the original additive one. The postulate of energy equivalence (Cordebois and Sidoroff, 1982; Carol et al., 2001) is employed to restore the major symmetry of the stiffness tensor. The classical PNP is performed on the strain tensor rather than the effective stress for the sake of numerical efficiency.

In this work, an alternative approach is proposed, in which the conceptual simplicity and numerical efficiency associated with the original additive bi-scalar damage model is preserved as much as possible. As we will show, the positive/negative components of a specific second-order tensor and the corresponding fourth-order PNP operators are not unique, unless an extra constraint is applied. Different supplementary condition yields rather distinct positive/negative components. In particular, the classical PNP scheme considered in the literature is only introduced in a heuristic manner for convenience. Therefore, it is possible to construct a novel PNP scheme such that the stiffness tensor of the original additive bi-scalar damage constitutive relation is always of major symmetry. Motivated by this viewpoint, the objective of this paper is fourfold: *(i)* to revisit the classical PNP scheme and to demonstrate its adverse effects on the damage model; *(ii)* to derive a novel variationally consistent PNP in energy norm and to give the explicit solution; *(iii)* to cast the additive bi-scalar damage model into the thermodynamics with internal variables; and finally, *(iv)* to illustrate its numerical performances in the modeling of concrete under static and seismic loadings.

The remainder of this paper is outlined as follows. Section 2 presents the original additive bi-scalar damage model. A generalized definition of the PNP is given with respect to the effective stress tensor. Section 3 addresses the classical PNP scheme and its variational interpretation. The adverse effects on the original additive bi-scalar damage model are then demonstrated. On the one hand, these theoretical analyses consolidate the classical PNP scheme heuristically considered in the literature. On the other hand, they also shed lights to other alternative PNP schemes. Section 4 is devoted to the novel variationally consistent PNP scheme in energy norm and the explicit solutions. In Section 5 the original additive bi-scalar damage constitutive relation is re-derived within the thermodynamics with internal variables. In particular, an energy release rate based damage criterion appropriate for tensile failure in concrete like quasi-brittle solids is established with no *ad hoc* assumption. The novel additive bi-scalar damage model is then validated in Section 6 with numerical examples of single-point verification as well as benchmark tests of concrete under both static and seismic loadings. The most relevant concluding remarks are drawn in Section 7, followed by three appendices closing this paper.

Notation. Compact tensor notation is used as much as possible. As a general rule, scalar a is denoted by a light-face italic minuscule (Latin or Greek) letter; vector $\boldsymbol{a}$, second-order tensor $\boldsymbol{A}$ and fourth-order tensor $\mathbb{A}$ are signified by boldface minuscule, majuscule and boldface majuscule letters, respectively. Operators ‘tr(·)’ and ‘$(\cdot)^{\text{sym}}$’ indicate the trace and sum-type symmetrized operators, respectively, defined as $\text{tr}A = A_{ii}$ and $(A^{\text{sym}})_{ij} = (A_{ij} + A_{ji})/2$. Symbols ‘·’ and ‘:’ denote the inner products with single and double contractions, respectively. The dyadic product ‘⊗’ and the symmetrized outer product ‘$\overline{\underline{\otimes}}$’ are defined as

$$(\boldsymbol{A} \otimes B)_{ijkl} = A_{ij}B_{kl}, \qquad (\boldsymbol{A} \overline{\underline{\otimes}} \boldsymbol{B})_{ijkl} = \frac{1}{2}\left(A_{ik}B_{jl} + A_{il}B_{jk}\right)$$

Letters $\boldsymbol{1}$ and $\mathbb{I} := \boldsymbol{1} \overline{\underline{\otimes}} \boldsymbol{1}$ signify the second-order and symmetric fourth-order identity tensors, respectively. The McAuley brackets $\langle x \rangle$ and Heaviside function $H(x)$ are defined as $\langle x \rangle = x$, $H(x) = 1$ if $x > 0$, and $\langle x \rangle = 0$, $H(x) = 0$ otherwise.

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