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A constitutive model for transversely isotropic material with anisotropic hardening

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Many materials such as wood, as well as rigid and flexible foams, exhibit transverse isotropic or even anisotropic mechanical properties in terms of their elastic and inelastic responses. However, there appear to be few constitutive models that are able to appropriately and accurately describe them. In this study, constitutive model that describes transversely isotropic material displaying anisotropic hardening, based on a weak form of definition of anisotropic hardening, is proposed. This form of hardening is incorporated into a modified yield function by introducing a matrix comprising dimensionless hardening functions. A modified stress is also defined, such that the yield surface remains stationary in the associated stress space, thus enabling the proposed anisotropic hardening function to be evaluated. The model is then used to predict the stress-strain curves and deformation of a transversely isotropic polyethylene foam exhibiting transversely isotropic hardening, subjected to compression at different angles to the axis perpendicular to the plane of isotropy. Good agreement between predictions based on the proposed model and Jebur's experimental data (Jebur, 2013) is observed; for comparison, predictions employing an earlier transversely isotropic model but with isotropic hardening (Tagarielli et al., 2005) shows noticeable deviation from the experimental data. The results substantiates the validity of the proposed anisotropic hardening model in describing the mechanical and deformation response of a transversely isotropic material exhibiting transversely isotropic hardening.

1. Introduction

It has been observed that many materials and structures such as wood (Gibson and Ashby, 1999), octet trusses (Deshpande et al., 2001), and rigid and flexible foams (Lee and Ramesh, 2004), exhibit transverse isotropy in their mechanical properties. Consequently, researchers have directed their attention at developing constitutive models to describe the mechanical response of transversely isotropic materials (Lubliner, 2008; Guo et al., 2008; Hill, 1998; Tagarielli et al., 2005). A well-recognized one for isotropic foam was proposed by Deshpande and Fleck (2000), who employed a quadratic description of the yield surface of metallic foam and obtained good agreement between experimental and theoretical results. This isotropic foam model was then further developed to describe the transversely isotropic elastic and initial yield characteristics of compressible solids (foams) (Tagarielli et al., 2005), and was successfully applied to balsa wood. However, in this model, the hardening behavior after initial yield was assumed to follow a scalar function, which does not capture anisotropic hardening. Some constitutive models have also been proposed for transversely isotropic materials, based on hyperelastic or viscoelastic theories; Jemiolo generalized Odgen's model for isotropic material to include transverse anisotropy under finite deformation (Jemioło and Telega, 2001); Peng developed an anisotropic hyper-elastic constitutive model for biological materials (Peng et al., 2006), while Zhurov proposed a transversely isotropic visco-hyperelastic description of compressible soft tissues (Zhurov et al., 2007). However, these models are not applicable to large inelastic deformation, since they are based on elasticity. For plastic deformation, Guo (Guo et al., 2008; Guo and Caner, 2010) developed a transversely isotropic model for porous materials based on a neo-Hookean strain energy function, while Jebur (2013) utilized a hyperfoam model to predict the compressive stress for polyethylene foam and obtained good results. However, in Guo's model and the hyperfoam model, the influence of loading direction was not considered. Hence, for these two models, the parameter values from fits with experiments for one loading direction are not applicable for loading in other directions.

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Apart from transverse isotropy of the elastic response and initial yield, post-yield hardening is generally anisotropic for transversely isotropic materials. Research into anisotropic hardening dates back to the 1950s (Besseling, 1953), but significant findings seem to have been published only after the 1970s. An anisotropic hardening constitutive model has been proposed by Mrŏz, based on introducing a set of nesting surfaces within a stress-space, to define how the hardening moduli vary during deformation (Mroz et al., 1978). He subsequently applied this model successfully to the analysis of elastic-plastic deformation of soil (Mroz et al., 1979). Later, Hashiguchi also developed an anisotropic hardening model by defining three surfaces — a loading surface, a sub-yield surface and a distinct-yield surface (Hashiguchi, 1981). Voyiadjis derived an Eulerian constitutive model for finite deformation plasticity with anisotropic hardening (Voyiadjis and Kattan, 1989), and more recently, a familiar constitutive model known as the homogeneous anisotropic hardening (HAH) model has been proposed by Barlat (Barlat et al., 2011, 2013, 2014). It was reported that the HAH model is currently the only one which can capture the Bauschinger effect adequately, together with the evolution of anisotropic hardening in sheet metals (Yoshida et al., 2015). As a result, the HAH model has been utilized by subsequent researchers in terms of incorporating a dislocation-based hardening model (Lee et al., 2013), developing a stress update algorithm (Lee et al., 2015) and application to high strength steel (Fu et al., 2016). However, the derivation and numerical implementation of the HAH model are complicated and has not yet been successfully applied to large deformation plasticity.

This work is directed at developing a simple, transversely isotropic constitutive model to capture anisotropic hardening under monotonic loading, using classical plasticity theory, with the objective of describing compressible materials such as rigid and flexible foams which exhibit transverse isotropy at initial yield, followed by anisotropic hardening. The transversely isotropic model by Tagarielli–Deshpande (Tagarielli et al., 2005) is first briefly described, before an anisotropic hardening model based on extending the Tagarielli–Deshpande model is elaborated on. Predictions base on the proposed anisotropic model are then compared with experimental data reported by Jebur (2013).

2. Phases of anisotropic response

Anisotropy, or direction-dependent mechanical properties, can be considered in terms of three phases: elasticity, initial yield, and post-yield hardening (or softening); the anisotropy of these phases generally differs. To aid understanding, the following definitions are first introduced before the constitutive model is presented.

Defenition 1. A material is anisotropic if its elastic behavior (i.e. Young's modulus, Poisson's ratio, etc) or initial yield stress is directionally dependent; it is otherwise isotropic. In the present work, materials which are transversely isotropic and orthotropic, also fall under Definition 1. For instance, a "transversely isotropic material" refers to a material exhibiting transverse isotropy in either its elastic behavior or initial yield response.

Defenition 2. Let Y_{ij} $(i, j = 1, 2, 3)$ be functions representing the flow stresses along the 1, 2 and 3 axis directions, and Y_{ij}^0 are the respective initial yield stresses for uniaxial loading and shear. The material exhibits isotropic/anisotropic hardening if the ratios of the six flow stresses to the initial yield stresses, defined by the hardening function $h_{ij} = Y_{ij}/Y_{ij}^0$ *(no sum on i, j;* $i, j = 1, 2, 3)$ are equal/unequal.

Fig. 1 depicts a two-dimensional case for Definition 2. Traditionally, isotropic/anisotropic hardening implies that the shape of the yield surface is preserved (Fig. 1(a)), or varies (Fig. 1(b)) during deformation. Based on this perspective of isotropic/anisotropic hardening, an infinite number of experiments are needed to identify the isotropic/anisotropic hardening behavior, because the evolution of all points on the yield surface should be determined to ascertain whether the shape of the yield surface is preserved or varies during its evolution. For practicality, Definition 2 in this study will adopt a "weak" form of the exact definition of isotropic/anisotropic hardening — i.e. instead of checking all the points on the yield surface, Definition 2 will only focus on specific directions — two uniaxial (Y_{11}, Y_{22}) and one shear Y_{12} — for a 2D case as shown in Fig. 1. For a general three-dimensional case, Definition 2 will examine the shape of the yield surface for six modes of loading (three uniaxial and three shear), based on the dimensionless hardening ratio functions h_{ij}. Since the dimensionless hardening functions h_{ij} are defined by nondimensionlizing the flow stress with respect to the initial yield stress, it will only reflect the anisotropy of the post-yield phase because the anisotropy of the initial yield stress is already excluded by the nondimesionlization.

3. Isotropic hardening model

3.1. Brief description of Tagarielli–Deshpande model

This effort to develop a constitutive model for transversely isotropic materials with anisotropic hardening behavior, is an extension of the work by Deshpande (Deshpande and Fleck, 2000; Tagarielli et al., 2005). In Tagarielli and Deshpande's work, a transversely isotropic compressible solid is described with respect to a Cartesian coordinate system (1,2,3), such that the directions 1-2 define the plane of isotropy, and direction 3 is the direction along which the mechanical properties differ. The relationship between the Cauchy stresses and the elastic strains is given by:

$$\boldsymbol{\sigma} = \boldsymbol{C}\boldsymbol{\varepsilon^e} \tag{1}$$

where the Cauchy stress $\boldsymbol{\sigma}$ and the elastic strain $\boldsymbol{\varepsilon^e}$ tensors follow the Voigt notation: $\boldsymbol{\sigma} = [\sigma_{11},\ \sigma_{22},\ \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]^T$, $\boldsymbol{\varepsilon^e} = [\varepsilon_{11}^e,\ \varepsilon_{22}^e,\ \varepsilon_{33}^e,\ 2\varepsilon_{12}^e,\ 2\varepsilon_{23}^e,\ 2\varepsilon_{31}^e]^T$, and $\boldsymbol{C}$ is the elastic stiffness matrix with the inverse:

$$\boldsymbol{C^{-1}} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{13}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & -\frac{\nu_{13}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_3} & -\frac{\nu_{13}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{12})}{E_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{E_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{E_{13}} \end{bmatrix}$$

where E_1 and E_3 are the Young's moduli in directions 1 and 3 respectively; E_{13} is the shear modulus in the 1–3 plane, and ν_{12} and ν_{13} are the two Poisson's ratios defined by $\nu_{12} = -\dot{\varepsilon}_{22}^e/\dot{\varepsilon}_{11}^e$, $\nu_{13} = -\dot{\varepsilon}_{33}^e/\dot{\varepsilon}_{11}^e$ for uniaxial loading along directions 1 and 3 respectively.

Lubliner (2008) analyzed the yield criterion for anisotropic materials, and according to his analysis which adopted the generalized von Mises criterion derived by Hill (1998), the effective stress for an anisotropic material can be expressed as:

$$\bar{\sigma}^2 = A_{ijkl}\sigma_{ij}\sigma_{kl} \tag{2}$$

where $\boldsymbol{A}$ is a 4th order tensor which has the same symmetry as the elastic stiffness tensor ($A_{ijkl} = A_{jikl} = A_{klij}$). For orthotropic materials with three mutually perpendicular planes of symmetry, only six constants are needed in tensor $\boldsymbol{A}$ (Hill, 1948), while for transversely isotropic materials, the number of constants will reduce to five, as shown by Tagarielli and Deshpande (Tagarielli et al., 2005).

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