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Influence of the local mass density variation on the fracture behavior of fiber network materials

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ABSTRACT

The fracture process in two fiber network materials, a low- and a high-density paper, is analyzed experimentally and numerically. The high-density paper is able to localize continued fracture to very small defects while a rather large defect is required in the low-density paper. Whereas the high-density paper has a homogeneous and limited variation in local mass density, the low-density paper is substantially more heterogeneous and has a higher local mass density variation. It is hypothesized that these fairly large regions of lower mass density govern the fracture process in paper and similar fiber network materials. A nonlocal fracture model is applied to describe and capture this length scale phenomenon and intents to simulate forces bridged over distant regions in the material via connected fibers. The suggested fracture hypothesis is consistent with experiments and hence offers an explanation to why network materials with different mass density variation may fracture differently.

1. Introduction

The fracture behavior of natural cellulosic fiber materials such as paper has been extensively studied during the past four decades (among others Seth and Page, 1974; Niskanen 1993; Considine et al., 2011; Mäkelä and Fellers, 2012; Hagman and Nygårds, 2012; Coffin et al., 2013; Isaksson and Hägglund, 2007; Niskanen et al. 2001). Low-basis-weight paper, commonly referred to as tissue paper, is widely used on a daily basis for a variety of household needs. Several physical properties distinguish them from printing and packaging papers including tensile response due to a more open sparse network structure offering greater flexibility. Paper essentially consists of a discontinuous stochastic network of cellulose fibers and is usually manufactured by dewatering a fiber-suspension on a wire. The fibers have an inherent capability to form bonds between them without any additives. Since the fibers are much longer than the thickness of the paper sheet, the fiber network is approximately planar. Typically, cellulosic fibers that are used to produce papers are 1–3 mm in length and 20–40 µm in cross-sectional width, depending on their origin and on the papermaking method. Cellulosic fibers are sensitive to the variations in environmental conditions such as relative humidity and temperature, affecting their dimensions and mechanical properties. As a consequence, the mechanical properties of papers also depend on their moisture content (Niskanen, 2011; Haslach, 2000).

Several studies have analyzed paper materials sensitivity to defects, e.g. Coffin et al. (2013) or Hägglund and Isaksson (2006), indicating that classical fracture mechanics theories have severe difficulties capturing the mechanical behavior of low-density paper materials. For high-density papers, a very small defect is sufficient to localize continued crack growth, as foreseen by classical fracture theories. For low-density papers, however, the situation is different because the fracture process seems unaffected by small defects and global final fracture does not necessary initiate at small defects. Moffat et al. (1973) reported that global fracture passes through regions in the paper having a local mass density (or grammage) below the paper's average. According to experimental and numerical studies, e.g. Wong et al. (1996) or Nazhad et al. (2000), local grammage and local strains are inversely proportional, meaning that high strains occur in regions of relatively low local mass density. These heterogeneities are often referred to as "flocs" and correspond to fiber aggregates in the network. The measurement of the spatial basis weight variation related to the flocs is hence essential since it may influence the fracture properties. However, estimation of the floc morphology in paper is not an easy task

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and there is no established standard, cf. Sampson (2001) or Ostoja-Starzewski and Castro (2003).

Because of the heterogeneous network structure, when a low-density paper is subject to mechanical load complex deformations arise and have to be captured properly, which puts high demands on the spatial resolution of a mechanical model. Linking the stress field on a heterogeneous subscale to a more homogeneous stress field on a scale above is mathematically difficult, especially if the subscale stress fluctuates and gradients are present (such as in the vicinity of crack tips, randomly positioned defects, pores, or regions of low mass density). Hence, the wide range of length scales affecting deformation and fracture in low-density papers poses a difficult modeling problem because the relation between the heterogeneous microstructure and the prevailing deformation processes leads to inherent size effects. Thus, if one aims at a material description above the scale of a discrete discontinuous substructure, i.e. within a macroscopic homogeneous model, it is clear that any fracture model of such materials has to capture length-scale effects from dominating subscales. One way to numerically analyze these fracture phenomena is to use high-resolved numerical models that capture details in the microstructure. Another possible way, explored in this study, is to use a nonlocal continuum model to incorporate length scales in the analysis (cf. Isaksson and Hägglund, 2009, 2013; Askes and Aifantis, 2011; Kröner, 1967; Aifantis, 2011; Eringen et al., 1977; Eringen, 2002).

2. Theory

How does the microstructure of a low-density paper affect its global fracture behavior? Traditionally, deformation and fracture analysis of papers have been performed using classical continuum theories, cf. Tryding (1996) or Niskanen (2011). However, in recent studies (e.g. Hägglund and Isaksson, 2006, 2009), it is reported that such continuum descriptions of low-density papers cannot fully describe deformations near a macroscopic crack mainly because microstructural effects alter the local strain and stress fields in the heterogeneous material, a phenomenon that commonly is referred to as length effects. Several techniques have over the years been suggested to extend traditional continuum formulations to include material-characteristic lengths. Often nonlocal continuum theories are applied that abandons Saint-Venant's principle of local action, i.e. the assumption that the mechanical state in a given point in a material is uniquely determined by the state in that point only as it includes interactions with its neighborhood. The nonlocal strategy is especially appealing for fiber network materials because long fibers links distant regions to each other and transfer forces over relatively long distances. Many different techniques have been proposed to incorporate nonlocality in continuum formulations (among others Askes and Aifantis 2011; Isaksson and Hägglund, 2013; Eringen et al., 1977; Gitman et al., 2010; Aifantis, 2014). A convenient approach is to compute a nonlocal stress tensor $\bar{\sigma}_{ij}$ in a point p as the weighted average of local stresses σ_{ij} in a surrounding domain Ω,

$$\bar{\sigma}_{ij}(p) = \Psi^{-1}\int_{\Omega}\phi(\rho)\sigma_{ij}(p')\mathrm{d}\Omega(p'),\ \Psi = \int_{\Omega}\phi(\rho)\mathrm{d}\Omega(p), \qquad (1)$$

where ρ is the distance between source and field points p and p', ϕ is an interaction kernel and Ψ a scaling factor that secure that the two stress tensors become equal for a homogeneous stress state in Ω. It is underlined that the use of (1) is here based on the physical argument of long-range mechanical interactions in the porous material via connected fibers. For simplicity, it is assumed that the paper is transversely isotropic on the macroscopic scale. (As will be discussed later, the paper is nearly transversely isotropic which substantially simplifies matters.) On the macroscopic scale, the material is considered homogeneous linearly elastic and a state of plane stress prevails. Then, consider a planar body containing a straight stationary semi-infinite crack, Fig. 1.

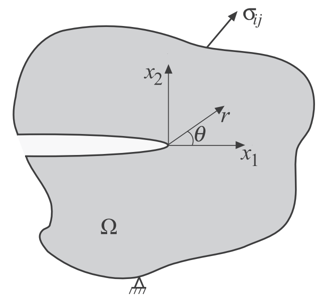

Fig. 1. Load and geometry on the macroscopic scale.

A Cartesian (x_1,x_2) and a polar coordinate system $(r = [x_1^2+x_2^2]^{1/2},\ \theta = tan^{-1}[x_2/x_1])$ are introduced with their origins coinciding with the crack-tip. The crack occupies the negative part of the x_1-axis, i.e. $x_1 < 0$ and $x_2 = 0$. Distant from the tip a pure opening mode field acts and the singular local stress tensor σ_{ij} is given by:

$$\sigma_{ij} = K_{\mathrm{I}}[2\pi r]^{-1/2} f_{ij}(\theta) \text{as } r \to \infty,\ i, j = 1, 2, \qquad (2)$$

where K_{I} is the mode I stress intensity factor (cf. Williams, 1956) and the angular function $f_{ij}(\theta)$ can be found in every book on fracture mechanics. Using (2) as source term in (1), the nonlocal stress $\bar{\sigma}_{ij}$ in the point (x_1,x_2) then takes the form

$$\bar{\sigma}_{ij}(x_1,x_2) = \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(x_1,x_2;x'_1,x'_2)\frac{1}{\sqrt{r'}}f_{ij}(\theta')\mathrm{d}x'_1\mathrm{d}x'_2. \qquad (3)$$

We aim for a simple but yet efficient numerical approximation of (3). Physical requirements of traction-free crack surfaces and finite stresses at infinity, together with an assumption of a Gaussian distributed kernel/weight function ϕ lead to:

$$\phi(x_1,x_2;x'_1,x'_2) = \begin{cases} \frac{1}{2\pi c^2}\exp\{-[(x_1-x'_1)^2+(x_2-x'_2)^2]/[2c^2]\} & \text{if } x'_1, x'_2 \cup \mathbb{Z} \\ 0 & \text{else} \end{cases} \qquad (4)$$

In (4), $\mathbb{Z}$ denote the set of field points (x'_1,x'_2) "visible" from the source point (x_1,x_2) and not separated by a discrete crack. The conditions in (4) secure that no unphysical interactions occur over a crack due to broken material connections. The material parameter c in (4) represents a characteristic length and controls the reach of long-range mechanical interactions in the material and hence reflects a dominant subscale. Moreover, using (3) and (4), the asymptotic normal stress at the tip, $\sigma_0 = \bar{\sigma}_{22}(0,0)$, is estimated to:

$$\begin{aligned}\sigma_0 &= \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\sqrt{r'}}\frac{1}{2\pi c^2}\exp[-r'^2/2c^2] \\ &\quad\times\cos^2\frac{\theta'}{2}\left[1+\sin\frac{\theta'}{2}\sin\frac{3\theta'}{2}\right]\mathrm{d}x'_1\mathrm{d}x'_2 \\ &= \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\int_0^{\infty}\frac{1}{2\pi c^2}\exp[-r'^2/2c^2]\sqrt{r'}\mathrm{d}r'\int_{-\pi}^{\pi} \\ &\quad\times\cos^2\frac{\theta'}{2}\left[1+\sin\frac{\theta'}{2}\sin\frac{3\theta'}{2}\right]\mathrm{d}\theta' \\ &= \frac{K_{\mathrm{I}}}{\sqrt{2\pi}}\frac{2^{3/4}\Gamma(\frac{3}{4})}{4\pi\sqrt{c}}\frac{24}{5}\end{aligned} \qquad (5)$$

where $\Gamma(\lambda) = \int_0^{\infty} s^{\lambda-1}\exp[-s]\mathrm{d}s$ is the Gamma function. Hence the nonlocal stress $\bar{\sigma}_{22}$ is finite at the tip and captures the value

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