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A constitutive model for the analysis of second harmonic Lamb waves in unidirectional composites

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ABSTRACT

In recent research on Structural Health Monitoring (SHM) guided waves, especially nonlinear Lamb waves, turned out to be a suitable means for monitoring material deterioration in thin-walled structures. In the corresponding numerical simulations on wave propagation the nonlinear elastic theory by Murnaghan is often implemented, which requires 14 material parameters for transversely isotropic materials. Enhancing an existing linear strain energy potential, a new nonlinear hyperelastic transversely isotropic material model is introduced which reduces the number of independent material parameters to six. In order to verify the applicability of the presented material model with respect to the simulation of nonlinear wave propagation in composite structures, and the generation of higher harmonic wave modes, the existence of a power flux from the fundamental to the higher harmonic mode is investigated analytically and numerically. Analytical considerations show that this power flux exists like in Murnaghan's theory. For the numerical validation the S_0–S_0 mode pair in the low frequency range is used. Therefore, the amplitude of the second harmonic wave mode is oscillating with increasing propagation distance. This behavior is in excellent agreement with the theoretical prediction. It is shown further, that even for an oscillating behavior the amplitude of the second harmonic mode can be approximated by a linear curve fit over a considerably propagation distance and hence shows a quasi cumulative behavior. Therefore, the introduced material model is an advantageous alternative to Murnaghan's theory to simulate the second harmonic Lamb wave generation due to in composite structures.

1. Introduction

In carbon fiber reinforced polymers (CFRP) fatigue damage starts at an early stage of lifetime (M. Kaminski et al., 2015). The micro-structural damage is growing during operation and may finally lead to a catastrophic structural failure (Harris, 2003). Therefore, the monitoring of composite material is essential even at an early stage. The observation of piezoelectrically excited Lamb wave with simultaneous generation of the second harmonic mode due to structural nonlinearity has proven to be an adequate technique. The detection of micro-structural damage was experimentally investigated mainly for isotropic material as presented in Bermes et al. (2007); Pruell et al. (2007); Pruell et al. (2009); Xiang et al. (2012) and Xiang et al. (2015) but also for composite structures (Li et al., 2012; Rauter and Lammering, 2015; Rauter et al., 2016). In general, the amplitudes of second harmonic modes are very small and hence, they subside very quickly. In order to overcome this shortcoming and to ensure an accurate extraction of the second harmonic amplitude the waves should be excited at a frequency at which a cumulative effect takes place (Deng, 1999; 2003; de Lima and Hamilton, 2005; Chillara and Lissenden, 2014; 2016; Müller et al., 2010), so that the second harmonic mode amplitude is linearly growing along the propagation distance.

To verify the experimental investigations and to analyze the further potential of this method numerical simulations are essential. Two different analytic descriptions for the second harmonic Lamb wave generation in thin-walled structures exist. In Deng (1999,2003) the method of nonlinear acoustic reflection at an interface and a modal analysis approach with a second-order perturbation approximation are introduced. It was found that the amplitude of the second harmonic mode grows linearly with the propagation distance, if the phase velocity of the primary and the second harmonic mode coincide. The second model presented in de Lima and Hamilton (2005) is based on the normal mode expansion technique and the reciprocity relation by Auld (1973); 1990). Beside the matching of the phase velocities a power flux condition is considered necessary for the existence of the secondary wave field. The power flux condition has been analyzed in more detail in Chillara and Lissenden (2014), Chillara and Lissenden (2016) and

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Müller et al. (2010) and numerical simulations for isotropic material are given in Liu et al. (2013c); 2013b), confirming the cumulative effect in case of matching phase velocities. However, if the phase velocities do not coincide the second harmonic amplitude shows oscillating behavior with the propagation distance, which can be described by a spatial periodicity. In Wan et al. (2016) it is shown that for an oscillating behavior of the amplitude a propagation distance can be determined for which the oscillating second harmonic amplitude can be approximated by a linear function and, therefore, shows a linear growth effect.

For the numerical simulation and the analytic description the nonlinear elastic theory by Murnaghan (1951) is used as material model so far. This hyperelastic nonlinear material model is based on a strain potential given by a third order Taylor series expansion (Blackburn, 1981). For isotropic material three third order elastic constants *l, m, n* (Murnaghan, 1951) and *A, B, C* (Landau and Lifshitz, 1959), respectively, describe the nonlinear material behavior beside two parameters from the linear theory of elasticity. These nonlinear material parameters represent the influence of mechanical stresses on the wave speed (Prosser, 1987). An extension of this material model to unidirectional and orthotropic material is given in Prosser (1987) and was recently successfully used to simulate the cumulative second harmonic Lamb wave generation in composite structures (Zhao et al., 2016). However, the nonlinear behavior of transversely isotropic material requires 14 material constants in total, five linear (second order stiffness coefficients) and nine nonlinear parameters (third order stiffness coefficients). Furthermore, due to their physical meaning, the determination of the nonlinear constants is very complex. Therefore, a new approach is presented in this study to simulate the second harmonic Lamb wave generation in unidirectional composite structures reducing the required amount of nonlinear material parameters.

Most of the established nonlinear material models for fiber reinforced composites represent biological tissue and, therefore, are based on the assumption of incompressibility. One example is the well-known Holzapfel-Gasser-Ogden model (Holzapfel, 2010) which is implemented in commonly used FE software by using the modified invariants. However, these formulations are not suitable for modeling composite structures, because in this case transversely isotropic material is described by only four independent material parameters (Vergori et al., 2013). This does not match with the linear description, which requires five independent coefficients. Hence, a new hyperelastic nonlinear transversely isotropic material model is introduced, which requires only six material parameters (five linear and one nonlinear) instead of 14 (five linear and nine nonlinear) for the nonlinear elastic theory to simulate the generation of second harmonic waves in composite structures. The obtained constitutive equation is able to describe the relevant features of nonlinear wave propagation in elastic solids even though same effects are not covered, e.g. the generation of longitudinal waves as second harmonics from shear waves. Therefore, first the strain energy potential is established and the power flux condition is evaluated analytically by using the procedure presented in Chillara and Lissenden (2016) and Müller et al. (2010). In a second step first the material model is validated by static and dynamic tests. Finally, the power flux condition is analyzed numerically by studying the behavior of the amplitude of second harmonic wave modes over the propagation distance.

The structure of this paper is as follows. Section 2 gives a brief overview of the theoretical foundation of hyperelastic material models and the nonlinear wave propagation. In Section 3, the approach for a nonlinear hyperelastic transversely isotropic material model is presented. Furthermore, the power flux condition is analyzed. After the validation in Section 4 the second harmonic mode generation is numerically simulated in Section 5. Thereafter, the representation of the nonlinear behavior is investigated in Section 6. Finally, Section 7 gives a summary and a conclusion of the presented work.

2. Theoretical foundation

2.1. Hyperelastic material models

For hyperelastic materials the stress state is assumed to be independent of the load path and thus solely determined by the strain state. Therefore, a potential function Ψ exists, which is called strain energy function. For isotropic material it can be written as a function of the principal strain invariants I_1, I_2, I_3 which can be expressed by the principal stretches λ_1, λ_2, λ_3, see (Malvern, 1969). The latter are commonly used in continuum mechanics and in context with the finite element method.

2.1.1. Basic equations

In the following, the right Cauchy-Green tensor

$$\mathbf{C} = \mathbf{F}^{\top} \cdot \mathbf{F} \tag{1}$$

is used as a rotation-independent deformation tensor obtained from the deformation gradient $\mathbf{F}$. This deformation tensor has three independent invariants for isotropic material behavior (Malvern, 1969)

$$I_1 = \operatorname{tr}\mathbf{C} \qquad I_2 = \frac{1}{2}\left[\left(\operatorname{tr}\mathbf{C}^2\right) - \operatorname{tr}(\mathbf{C})^2\right] \qquad I_3 = \det\mathbf{C}. \tag{2}$$

In the case of transversely isotropic material, five invariants exist due to the dependence of the strains on the fiber orientation. In addition to the above mentioned invariants two pseudo invariants are defined which are given by Spencer (1984)

$$I_4 = \mathbf{a}_0 \cdot \mathbf{C} \cdot \mathbf{a}_0 \qquad \text{and} \qquad I_5 = \mathbf{a}_0 \cdot \mathbf{C}^2 \cdot \mathbf{a}_0, \tag{3}$$

where $\mathbf{a}_0$ is the vector of the fiber direction.

Based on the energy function Ψ the second Piola-Kirchhoff stress tensor $\mathbf{S}$ and the stiffness tensor $\mathbb{C}$ are obtained by

$$\mathbf{S} = 2\frac{\partial \Psi}{\partial \mathbf{C}} \qquad \text{and} \qquad \mathbb{C} = 4\frac{\partial^2 \Psi}{\partial \mathbf{C}^2}, \tag{4}$$

respectively. The derivatives of the invariants with respect to the right Cauchy-Green tensor are calculated as

$$\frac{\partial I_1}{\partial \mathbf{C}} = \mathbf{I} \qquad \frac{\partial I_4}{\partial \mathbf{C}} = \mathbf{a}_0 \otimes \mathbf{a}_0$$
$$\frac{\partial I_2}{\partial \mathbf{C}} = I_1\mathbf{I} - \mathbf{C} \qquad \frac{\partial I_5}{\partial \mathbf{C}} = \mathbf{a}_0 \cdot \mathbf{C} \otimes \mathbf{a}_0 + \mathbf{a}_0 \otimes \mathbf{C} \cdot \mathbf{a}_0$$
$$\frac{\partial I_3}{\partial \mathbf{C}} = I_3\mathbf{C}^{-1}.$$

Furthermore, due to the symmetry of $\mathbf{C}$ the derivates of $\mathbf{C}$ and $\mathbf{C}^{-1}$ with respect to $\mathbf{C}$ are given by Kaliske (2000)

$$\frac{\partial \mathbf{C}}{\partial \mathbf{C}} = \mathbb{I}^s = \frac{1}{2}\left(\delta_{ik}\delta_{lj} + \delta_{il}\delta_{jk}\right)\, \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l, \tag{6}$$

$$\frac{\partial \mathbf{C}^{-1}}{\partial \mathbf{C}} = -\frac{1}{2}\left(\mathbf{C}^{-1}_{ik}\mathbf{C}^{-1}_{lj} + \mathbf{C}^{-1}_{il}\mathbf{C}^{-1}_{jk}\right)\, \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l. \tag{7}$$

2.1.2. Linear transversely isotropic hyperelastic material model

A strain energy function which represents linear transversely isotropic material behavior can be formulated by use of the five invariants of $\mathbf{C}$. In terms of the Green-Lagrange strain tensor $\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I})$ this strain energy function is given by Reese et al. (2001)

$$\Psi_E = \frac{1}{2}\lambda(\operatorname{tr}\mathbf{E})^2 + \mu_T \operatorname{tr}\mathbf{E}^2 + \alpha(\mathbf{aEa})\operatorname{tr}\mathbf{E}$$
$$+ 2(\mu_L - \mu_T)(\mathbf{aE}^2\mathbf{a}) + \frac{1}{2}\beta(\mathbf{aEa})^2, \tag{8}$$