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Sliding cable modeling: An attempt at a unified formulation

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ABSTRACT

Sliding cable structures are systems where cables experience a relative sliding motion with other structural elements. The variety of structural systems using sliding cables led to a great diversity and scattering of the modeling approaches. This paper presents original developments expanding and generalizing the existing works and proposes a multi-node sliding cable model accounting for friction, with a general dynamic formulation, an effective numerical implementation and applicability to various material behaviors. General sliding equations are formulated, along with the unstretched length conservation constraint. Closed-form expressions of the Newton–Raphson scheme are developed to solve the sliding equations analytically while enforcing the conservation constraint. The formulation and its implementation are validated against a theoretical dynamic sliding cable mechanism and simulation results agree perfectly with the analytical solutions. The model is used to perform a parametric study of a complex system of sliding cables under dynamic loading. These simulations highlight the influence of the investigated parameters and prove the robustness and versatility of the proposed model over the existing ones from the literature.

1. Introduction

Cable systems are widespread in civil engineering and mechanical structures such as bridges, guyed structures, catenaries and power lines (Irvine, 1981). One of the main advantages of cables resides in their combination of high strength, lightness and flexibility. Cables can easily redirect and modify tensions in lifting machines thanks to pulley systems, they can also balance forces in supporting and suspension structures by allowing for changes in their geometry and mass distribution. In these systems, cables experience relative sliding motion with other structural elements such as pulleys, drums or sliding components. Specific studies of these sliding cables have been performed for various engineering applications such as electrical transmission lines (Aufaure, 1993, 2000), cranes and suspended cable systems (Dupire et al., 2015; Ju and Choo, 2005; Wang and Rega, 2010), suspended roofing systems (Chen et al., 2010; Hincz, 2009) and tensioned fabric membranes (Dinh et al., 2016; Pargana et al., 2010), protection structures (Boutillier, 2004; Ghoussoub, 2014; Nicot et al., 2001; Volkwein, 2005) and parachute systems (Zhou et al., 2004). These different works all address the mechanics of cables passing through pulleys or of sliding elements traveling along cables, but the scope of each of these studies is limited to some areas of analysis. The numerous approaches make different assumptions and use different formulations regarding four principal characteristics of sliding cables: the number of consecutive sliding elements, the account of friction, the type of analysis (static or dynamic) and the constitutive behavior of the cable material.

Many of the existing works are concerned with the sliding of a single element and use a 3-node model for that purpose: a central sliding node comprised between two end nodes. That is the case of the FEM models of Aufaure (1993, 2000) and Zhou et al. (2004), as well as the 'sliprings' elements of the LS-DYNA software (LSTC, 2006). The models proposed by Aufaure (1993, 2000) only allow sliding bounded between the 2 end nodes while the other models (LSTC, 2006; Zhou et al., 2004) suggest remeshing algorithms to allow continuous sliding on longer distances. To treat longer cable spans with multiple rest or lift points, multi-node models have been developed, in particular by Boutillier (2004), Chen et al. (2010), Ghoussoub (2014), Hincz (2009), Ju and Choo (2005) and Volkwein (2005). Such models provide effective and integrated formulations for systems containing multiple sliding nodes instead of using an assembly of several single-node models.

Whether single-node or multi-node, the previously introduced models do not all account for friction between the cable and sliding elements. The 3-node models from Aufaure (1993, 2000) and Zhou et al. (2004) as well as the multi-node models from

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(Chen et al., 2010) and Ghoussoub (2014) assume uniform tension over the cable, resulting in a frictionless and permanent motion of the sliding nodes along the cable. Studies focusing on belts and pulleys almost always account for friction (Childs, 1980; Ravikumar and Chattopadhyay, 1999) and Ju and Choo (2005), as well as Dupire et al. (2015), also argue that when applied to sliding cables, frictionless models can yield unrealistic and incorrect results. The account of friction appears to be critical and largely conditions the complexity of the modeling. Friction induces differences in tensions on each side of a sliding node. Models accounting for friction make different assumptions and use various numerical methods to treat it. Volkwein (2005) uses the uniform tension assumption around sliding nodes before applying a tension reduction on one side to account for friction. This method is simple and efficient but does not provide a realistic distribution of tensions in the cable. Most models accounting for friction consider non-uniform tension on both sides of sliding nodes and use a sliding criterion, based on a sliding function, to determine whether the nodes stick or slip. When the sliding function exceeds a given limit, sliding occurs and tensions are balanced through a given sliding mechanism in order to match the sliding limit.

Different sliding criteria can be found (Boutillier, 2004; Hincz, 2009; Ju and Choo, 2005) as well as different numerical resolution schemes. For single-node models, usual root-finding algorithms can be used. For multi-node models, different strategies exist. Ju and Choo (2005) build a cable super-element considering slip around sliding nodes as a part of the elastic deformation of the adjacent sub-elements. Hincz (2009) uses a fixed step relaxation algorithm, thus avoiding the formulation of a multivariate system of equations to be solved. This light and compact formulation suffers from a great computational cost and potential excesses of the sliding limit due to the fixed step search. Boutillier (2004) defines a linearized multivariate problem that shows better performances but can only be applied for static or quasi-static analyses.

In general, existing models are developed for either static or dynamic analyses. Most of the existing models are only formulated for statics (Aufaure, 1993; 2000; Chen et al., 2010; Dinh et al., 2016; Dupire et al., 2015; Ju and Choo, 2005; Nicot et al., 2001; Wang and Rega, 2010). Some are developed for specific non-linear static applications by means of low-speed dynamic (Boutillier, 2004; Ghoussoub, 2014) and dynamic relaxation (Hincz, 2009) algorithms. Fewer works are dedicated to dynamic problems (LSTC, 2006; Volkwein, 2005; Zhou et al., 2004).

As for the cable material properties, all of the existing sliding cable models, except from LSTC (2006) that is used in Erhart (2012) with Hill's muscle model, are formulated for linear elastic material only. This assumption is often built-in the mathematical formulation of the models. It may also be a mandatory condition for the resolution method to function properly, thereby limiting the potential applications of such models to a more realistic, non-elastic and non-linear cable material behavior.

None of the previously introduced developments present the modeling capabilities to combine these four properties (multi-node, friction, dynamic analysis and general constitutive behavior of the cable material) altogether. In this article, a unified sliding cable model is proposed. This model aims at expanding the various existing approaches and consists of a multi-node sliding cable accounting for friction, with a general dynamic formulation, an effective numerical implementation and applicability to various material behavior. First, the mechanical concepts and the mathematical formulation for the dynamic sliding problem are introduced. The multi-node sliding equations derived from this mathematical analysis are solved under a conservation constraint using an analytically formulated Newton–Raphson scheme that ensures both respect of the constraint and computational efficiency. The model is then validated against the analytical solutions of a highly non-linear dynamic system. Eventually, applications to the modeling of the 'curtain effect', a complex dynamic sliding process taking place in flexible protection structures, is conducted to investigate the full capabilities of the model.

2. General sliding cable formulation

2.1. Definition of the sliding cable system

The cable is defined as a set of consecutive segments that can be fully described by the position of the segment vertices, which is a set of ordered nodes located at the end points of the segments. The nodes are broken down into two types: sliding nodes and non-sliding nodes (Fig. 1). Non-sliding nodes can represent either discretization nodes of the cable matter itself or elements firmly anchored to the cable, there is no relative motion between the cable and non-sliding nodes. As a result, the arrangement of non-sliding nodes along the cable is invariable. The two end nodes of sliding cables must be non-sliding nodes. Sliding nodes represent external elements either free to move along the cable or around which the cable moves, there is a relative sliding motion between the cable and sliding nodes. Such sliding elements can represent pulleys, drums, shackles or any external element having a relative sliding motion with the cable. Assuming that these elements cannot go past each other, the arrangement of successive sliding nodes along the cable is also invariable. The overall arrangement of all the nodes may however change. Sliding nodes may experience continuous sliding over several segments, thereby going over non-sliding discretization nodes and modifying the order of the consecutive nodes. These permutations in the consecutive order of the nodes must be performed by remeshing algorithms as discussed in the introduction.

Stress and strain are constant over each segment. The uniaxial strain ε is expressed in terms of the current length l and unstretched length l_0 of a segment using Cauchy strain as $\varepsilon = (l - l_0)/l_0$. The expression of the stress σ depends on the constitutive relation used for the cable material behavior. Tension in the segments is preferred over stress for convenience in structural applications. Assuming a constant cross-section surface area A, tension can be expressed as $T = \sigma A \geq 0$. Tension must remain positive as the cable is considered unable to resist axial compression forces.

2.2. Sliding criterion and mechanism

Considering a perfectly flexible circular arc of cable subject to tensions T_A and T_B at its ends such as $T_B \geq T_A \geq 0$ and to friction along the arc length (Fig. 2), the static equilibrium of the cable yields the capstan equation (Childs, 1980)

$$T_B - T_A \mathrm{e}^{\mu\alpha} = 0 \quad (1)$$

where μ is the friction coefficient between the cable and the circular element and α is the total wrap angle. The assumption that cables are perfectly flexible theoretically limits the validity of the capstan equation to cables with small flexural rigidity and large curvature. For smaller curvature, the assumption of a perfectly flexible cable remains acceptable when the wrap angle is limited. In practice, mechanical systems using cables bent with small curvature and large wrap angles are designed so that the cables are flexible enough to operate without resisting extensive bending. Modifications of the capstan equation to account for flexural rigidity of the cable can be made (Jung et al., 2008) but are not of interest herein.

Eq. (1) defines the sliding limit that corresponds to the limit state equilibrium, meaning that no value of T_A and T_B can verify $T_B - T_A \mathrm{e}^{\mu\alpha} > 0$. The sliding function can then be defined as:

$$S(T_A, T_B) = T_B - T_A \mathrm{e}^{\mu\alpha} \leq 0 \quad (2)$$