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An efficient model for the frictional contact between two multiferroic bodies

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ABSTRACT

This paper presents a semi-analytical model (SAM) for three-dimensional frictional magnetoelectroelastic (MEE) contact of two multiferroic bodies, together with a set of effective solution methods. The frequency response functions (FRFs) for the MEE fields in a multiferroic half-space are analytically derived with respect to a unit concentrated normal force, a unit concentrated tangential force, a unit electric charge, and/or a unit magnetic charge, which are then converted into the results of continuous Fourier transforms of the influence coefficients (ICs), followed by the discrete Fourier transforms with a proper aliasing treatment. The conjugate gradient method (CGM) is used to obtain the unknown distributed pressure. Furthermore, the discrete convolution-fast Fourier transform (DC-FFT) algorithm is implemented to calculate the in-plane electric/magnetic potentials and subsurface stresses. The model is implemented to analyze the frictional sliding contact between a half-space and a sphere, and to study the coupled effects of surface electric/magnetic charges and friction on contact behaviors, including pressure, stresses, and electric/magnetic potentials. A sensitivity analysis is also conducted to evaluate the influences of friction and material properties on the contact-induced multifield coupling behaviors. A number of case studies are committed, and the results indicate that electric/magnetic charge densities and the friction coefficient strongly influence the contact pressure, stress, and electric potential.

1. Introduction

Multiferroic materials permit coexistence of at least two ferroic orders, ferroelectricity, ferromagnetism, or ferroelasticity. These materials have demonstrated attractive multifunctions valuable to intelligent systems, such as magnetostrictive transducers, ultra-sensitive magnetometers, and electronic instrumentations (Nan et al., 2008; Wang et al., 2010a, Ma et al., 2011; Fiebig et al., 2016). When the multiferroic materials are in contact, their performances are under the influence of magnetoelectroelastic (MEE) coupling. The investigation of the MEE contact behaviors of the multiferroic materials is the first step towards in-depth understanding of the material interfaces (Chen et al., 2010; Zhou and Lee, 2013; Rodríguez-Tembleque et al., 2016a) and their overall responses to different physical loadings, which is needed to guide the optimal design of the material structures and the experimental planning to evaluate and characterize these materials (Prashanthi et al., 2011). However, the contact analyses involve complicated physical properties of the materials and field coupling, requiring a huge amount of computation. Therefore, an efficient modeling approach is highly demanded.

Analytical studies have been committed to investigate the contact behaviors of piezoelectric materials (Ding, 1996; Hou et al., 2003; Ding et al., 2000; Giannakopoulos and Suresh, 1999). The research of Hou et al. (2003) was among the first efforts to study the contact of transversely isotropic multiferroic bodies, inspired by the work reported in (Puja, 1997) for the contacts of transversely isotropic materials based on the potential theory (Fabrikant, 1989). They obtained exact solutions for transversely isotropic multiferroic materials in elliptical Hertzian contacts. Problems involving axisymmetric indenters, such as a flat-ended circular punch and a sphere and circular cone, were addressed in (Chen et al., 2010), where the transversely isotropic multiferroic half-space indented by either an insulating or a conductive indenter was considered. Similarly, Li et al. (2014) presented a set of fundamental solutions for the frictionless contact of a multiferroic half-space and a semi-infinite rigid punch. Likewise, the solution to the elliptical indentation in MEE fields was also obtained (Li et al., 2015). The above-mentioned works were more focused on frictionless

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Nomenclature

a	contact radius, m
a_{cgm}	step length in CGM
a_0	contact radius in elasticity, m
a_1, a_2, a_3	variables relating to material properties
a_{41}, a_{51}, a_{61}, a_{71}	variables relating to material properties
a_{42}, a_{52}, a_{62}, a_{72}	variables relating to material properties
a_{43}, a_{53}, a_{63}, a_{73}	variables relating to material properties
A_0, A_1, A_2, A_3, A_4	unknowns in potential functions
B_x, B_y, B_z	magnetic induction along the *x, y, or z* direction, N/(Am)
c	a group of variables relating to material properties
c_{ij}	elastic moduli, 10^9 N/m^2
C^{ux}, C^{uy}, C^{uz}, C^{φ}, C^{ψ}, C	influence coefficients
d_{ij}	piezomagnetic coefficient, N/(Am)
d_{cgm}	descent direction in CGM
D_x, D_y, D_z	electric displacement, C/m^2
e_{ij}	piezoelectric coefficient, C/m^2
f	selected material constant
f^0	f 's reference value
g	gap between two surfaces, m
g_{ij}	electromagnetic coefficient, C/(Am)
g_b	surface magnetic charge density, N/Am
G^{ux}, G^{uy}, G^{uz}, G^{φ}, G^{ψ}, G	frequency response functions
G_b	total magnetic charge, Nm/A
h_0	initial separation between two surfaces, m
i	imaginary unit
k_{ij}	variables relating to material properities
L	length of cylindrical punch or flat-ended rectangular punch, m
m, n	Fourier-transformed variables with respect to *x, y* directions, respectively
n_0, n_1, n_2, n_3, n_4	variables relating to material properties
M, N	mesh numbers along the *x, y* directions
M_e, N_e	refined mesh numbers
p_x	shear tractions parallel to the *x* direction, Pa
P_x	applied tangential force along the *x* direction, N
p_z	pressure, Pa
P_z	applied normal force, N
q_b	surface electric charge density, C/m^2
Q_b	total electric charge, C
R	radius of a spherical punch, m
R_x, R_y	Major and minor radius of an ellipsoidal punch, m
R_L	radius of a cylindrical punch, m
R_1, R_2	inner and outer diameter of a flat-ended cylindrical shell, m
s_j	variables relating to material properties
t_1, t_2,..., t_{12}	variables relating to material properties
T_1, T_2, T_3	intermediate matrixes
u_x, u_y, u_z	displacements along the *x, y, z* directions, m
V	contact behavior for u_z, u_x, ϕ, φ
x, y, z, z_j	Cartesian coordinates in the spatial domain
Y	shape function
Z_0	height of rough surface, m

Greek letters

α	distance of a node, (m, n), measured from origin of the frequency domain
α_1, α_2, α_3	variables relating to material properties
β_{ij}	variables relating to material properties
γ	refinement level
δ	indentation depth, μm
ε_{ij}	dielectric permittivity, 10^{-9}C^2/(Nm^2)
μ_{ij}	magnetic permeability, 10^{-6}Ns^2/C^2
μ	friction coefficient
ρ_1, ρ_2, ρ_3	variables relating to material properties
σ_{ij}	stress components, Pa
σ_s	von Mises stress, Pa
ϕ	electric potential, V
φ	magnetic potential, A
ω_{ij}	variables relating to material properties
Γ_c	contact zone
Δ_x, Δ_y, Δ_z	grid size in the *x, y, z* directions, m

Special marks

≈	double continuous Fourier transform
^	discrete Fourier transform
-	surface variables
| • |	determinant of a matrix
IFFT	inverse fast Fourier transform

contacts. As for frictional contacts, many researchers studied the problems of a rigid punch sliding on a multiferroic body (Zhou and Lee, 2013; Elloumi et al., 2013; 2014; Kim, 2014; Zhou and Zhong, 2014). By using the Fourier integral technique, such contact problems can be resolved in Cauchy integral equations in terms of the unknown contact pressure to be solved numerically. Note that this method is convenient in a two-dimensional framework. Following this approach, Ma et al. (Ma et al., 2015) studied the frictional contact of a functionally graded multiferroic material with a flat punch, and issues related to frictional heating were further considered in their more recent work (Ma et al., 2016).

The theoretical works mentioned above were mainly focused on rigid punches and simple loading conditions, as well as idealized geometries (two dimensional or axisymmetric cases), which confine the application range of their solutions. Contacts encountered in engineering systems are subjected to more complex three-dimensional (3D) structures and loading conditions. The study on the contacts between two 3D deformable bodies should offer more general solutions and more accurate understanding to real-world problems. However, little has been done to solve general 3D MEE contact problems. Michopoulos et al. (2012), (2014), (2015) modeled the contacts of multiferroic materials in the framework of the finite element method (FEM). Recently, Rodríguez-Tembleque et al. (2016a) studied the frictional contacts between a rigid sphere and a multiferroic half-space by the boundary element method (BEM) in which only the surface domain was discretized. These investigations explored the possibility to utilize advanced numerical formulations.