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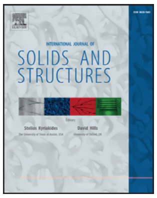

Anisotropic elastic-plastic deformation of paper: Out-of-plane model

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ABSTRACT

Laminated paperboard and paper is widely used in packaging products. Its out-of-plane properties play a crucial role in converting paperboard to a carton through the creasing and folding process. The aim of this study is to describe the out-of-plane mechanical response with an elastic-plastic model based on the observed experimental behavior. The model was derived from a thermodynamic framework and was based on the multiplicative split of the deformation gradient in the context of hyperelasticity. A structural tensor-based approach was applied to model the elastic deformation, while a multi-surface based yield criterion was adopted to describe the plastic behavior. The model considers both material densification and internal friction effects, which were observed experimentally. According to the experimental behavior, a quadratic yield locus under pure shear loading was assumed in the model. Finally, the model was validated with a cylinder compression test and found to capture the highly anisotropic, elastic-plastic behavior accurately.

1. Introduction

Laminated paperboard, widely used in packaging products, is commonly produced as a light-weight, multi-layer sandwich structure. In industry, almost all paperboard packaging includes a creasing operation to obtain crease-lines and a subsequent folding to form the final box. A good crease introduces delamination in the deformed zone to locally reduce its bending stiffness, which increases the ease of folding later on. The delamination is mainly dependent on both interface properties between different layers and out-of-plane properties of individual layers. Consequently, it is essential to capture these two aspects in order to obtain an accurate prediction of creasing and folding: the out-of-plane properties of individual layers and the interface delamination between different layers. The cohesive zone model (CZM) has been shown to provide a reasonable prediction of paperboard delamination behavior, and was therefore adopted by many researches, see for example Beex and Peerlings (2012); Nygårds (2009); Huang et al. (2014). To enable CZMs, studies have been conducted to characterize paperboard interface properties, e.g. Fellers et al. (2012); Korin et al. (2007); Nygårds et al. (2007); Li et al. (2016b; 2016a).

The second aspect which plays a significant role in converting paperboard to the final box is the out-of-plane behavior of an individual paper layer. Many studies have shown that it is reasonable to develop the in-plane and out-of-plane mechanical models in an uncoupled way, see e.g. Stenberg (2003); Xia et al. (2002). While there are many in-plane models for paper available (Xia et al., 2002; Borgqvist et al., 2014; Linvill and Östlund, 2016; Hagman and Nygårds, 2017; Bosco et al., 2016; Nygårds and Sundström, 2016), modeling of the out-of-plane behavior, particularly in a large deformation framework, is still limited and undeveloped. It is difficult to develop such models for several reasons. One is due to the lack of experimental data in the out-of-plane direction. Because of the small thickness of a single paper layer, performing accurate out-of-plane tension and shear measurements is a challenging task. In particular, the heterogeneous microstructure of paper leads to high deviation between experimental measurements. A second reason is that in-plane material models are not suitable in describing the out-of-plane mechanical behavior. Each paper layer is almost a two-dimensional fiber network, where very few fibers are orientated along the thickness direction (ZD). Consequently, the paper behaves quite different in the out-of-plane direction in contrast to the in-plane direction. Examples of such differences include the large deformation range, the densification effect and the internal friction effect, which are all more significant in the out-of-plane direction.

The out-of-plane mechanical response of paperboard has been experimentally studied by Stenberg et al. (2001a; 2001b); Stenberg and Fellers (2002) using a modified Arcan device. The experimental data indicate negligible in-plane deformation dur-

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ing the ZD loading tests. Additionally, the failure stress in out-of-plane shear was strongly dependent on the ZD compression of the material (internal friction effect). To capture these experimental observations, Stenberg (2003) proposed an elastic-plastic model based on the concept of bounding surface. Nygårds et al. (2007); Nygårds (2008) further experimentally studied the out-of-plane properties of paperboard using a double-notch shear (DNS) specimen. More recently, Hämäläinen et al. (2017) presented a numerical method to determine the out-of-plane shear moduli from the short span compression tester (SCT). For modeling the out-of-plane behavior, Tjahjanto et al. (2015) modified the approach proposed by Xia et al. (2002) to incorporate viscoelastic-viscoplastic effects. This model combined isotropic and kinematic hardening with back-stress terms and incorporated the ZD compression densification effects both in the out-of-plane elastic and plastic models. Also included in this model was the internal friction effect. For the ease of parameter calibration, the magnitude of the back-stress was defined the same as the hardening resistance of the opposite sub-surface. Girlanda et al. (2016) used this model to simulate the responses of the high-density cellulose-based fiber mats under two types of transient loading: compressive creep and stress relaxation. A similar multi-surface based elasti-plastic model was developed to analyze the deep drawing of paperboard by Wallmeier et al. (2015).

All the aforementioned constitutive models were developed in the small deformation framework, although large deformation occurred in the out-of-plane direction, especially as a result of ZD compression. Harrysson and Ristinmaa (2008) proposed a large strain elastic-plastic model based on the Tsai–Wu failure criterion. They also considered the plastic deformation of the material substructure by introducing an additional multiplicative split of the substructure. Recently, Borgqvist et al. (2015) modified the model from Xia et al. (2002) to take into account the distortional hardening in the finite elastic-plastic deformation framework. In this model, an additional set of coupling parameters were introduced to consider the interaction between different sub-surfaces during the hardening evolution. This model considered the internal friction effect and has been used to predict the occurrence of localized bands in both paperboard short-span compression and folding test (Borgqvist et al., 2016). However, most investigations into the paperboard creasing, folding and cutting processes have used comparatively simple yield criterions generally available in commercial codes or empirical formulas to describe the onset of yield and plastic flow, e.g. in Nagasawa et al. (2003); Beex and Peerlings (2012); Li et al. (2014); Huang et al. (2014).

When considering the material densification effect, not all the aforementioned models have been proven to be thermodynamically consistent. Additionally, most investigations using tests such as the combined compression-shear tests have focused on the plastic yielding along MD and CD under different compressive load levels. Since creasing and folding can occur in other directions as shown in Fig. 1, it is essential to evaluate the corresponding mechanical response. Therefore, the combined compression-shear tests were conducted along different directions. In the current work, a general out-of-plane elastic-plastic constitutive model was proposed to incorporate both material densification and internal friction effects in the finite deformation range. The model was derived from a thermodynamic framework and was based on the multiplicative split of the deformation gradient in the context of hyperelasticity. The elastic deformation was modeled by adopting the structural tensor concept, while the plastic evolution was described using multi-surface based yield criterion and a flow rule. Further, a straightforward calibration procedure was presented to identify the material parameters in the elastic as well as the plastic part. Finally, with the fitted parameters, a cylinder compression test was used to validate the model by comparing the obtained strain fields as well as the cylinder force-displacement curves.

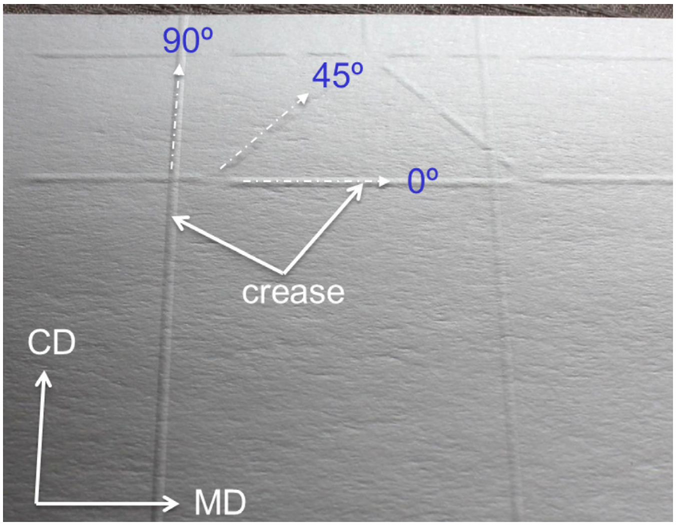

Fig. 1. Sample of creased paperboard during carton production (MD – machine direction, CD – cross machine direction).

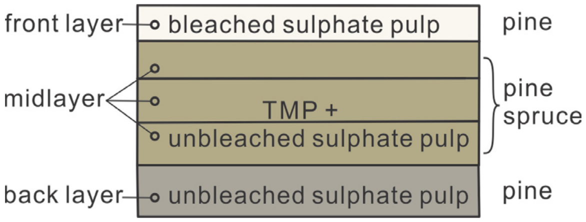

Fig. 2. Schematic of the paperboard sample (Only the midlayer was used in testing).

2. Experimental investigation

2.1. Paper material and experimental methods

The paper sample under consideration consists of three layers, as shown in Fig. 2. The in-plane mechanical properties have already been studied experimentally and numerically (Li et al., 2016a). Both top and bottom layer are made of pure pine sulphate pulp, whereas the midlayer is a mixture of unbleached softwood sulphate and Thermo-Mechanical Pulp (TMP). In order to consistently conduct tests on the same type of paper layer, the front and back layers were separated from the whole paperboard by grinding. The grinding process also removed the surface unevenness, which influenced measured curves in compression related tests significantly (Stenberg et al., 2001a). The final paper sample had a thickness of 0.21 ± 0.01 mm. The test samples were cut into 20 mm × 20 mm sheets to include a sufficient amount of fibers.

To characterize the mechanical behavior of paper, three kinds of tests were conducted in the out-of-plane direction: tension, compression, and combined compression-shear. The experimental setup is shown in Fig. 3. It includes a ‘ZWICK Z5’ testing machine to record the force and an ’ARAMIS 4M’ digital image correlation (DIC) system to capture the deformation. All experiments were performed in the same environmental conditions as in storage, with the room temperature being between 20° and 25°, and the ambient relative humidity being between 55% and 60%. The influence of the moisture content and temperature on paper was neglected in the current study. The experimental procedure for the out-of-plane compression test reported by Girlanda et al. (2016) was adopted. A