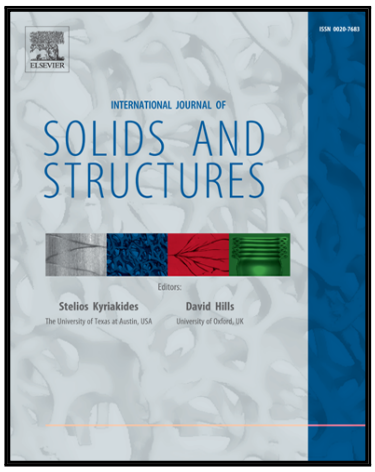

Three scales asymptotic homogenization and its application to layered hierarchical hard tissues

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Three scales asymptotic homogenization and its application to layered hierarchical hard tissues

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Abstract

In the present work a novel multiple scales asymptotic homogenization approach is proposed to study the effective properties of hierarchical composites with periodic structure at different length scales. The method is exemplified by solving a linear elastic problem for a composite material with layered hierarchical structure. We recover classical results of two-scale and reiterated homogenization as particular cases of our formulation. The analytical effective coefficients for two phase layered composites with two structural levels of hierarchy are also derived. The method is finally applied to investigate the effective mechanical properties of a single osteon, revealing its practical applicability in the context of biomechanical and engineering applications.

Keywords: Multiple scales, homogenization, hierarchical composite.

1. Introduction

Several biological and man-made materials exhibit a hierarchical structure at more than two length scales. The fascinating properties of biological composites are largely due to their unique structures, which are thought to be intimately related to the hierarchical and functional relationships between each of the scales ([1]). There exist several hierarchical materials in nature, including lotus leaves, kidney's glomerules, bones, etc. Applications of hierarchical composites include, but are not limited to, tissue engineering of biomimetic artificial tissues, drug delivery in healthy and malignant tissues, structural design ([1, 2, 3, 4]).

Modeling is a useful tool for predicting the effective behavior of heterogeneous media with periodic structure. Several analytical and computational models have been proposed to cal-